Capital Reallocation and Private Firm Dynamics

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ABSTRACT

This paper develops a theory of private firms whose primary business capital is internally created, cannot be rented, and is reallocated through infrequent, indivisible sales. The equilibrium allocation is efficient but exhibits dispersion in marginal products, returns, and transferable shares of business value. U.S. tax data are used to estimate technology parameters and evaluate tax reforms. An undistorted tax system uniformly taxes labor income, business income, and realized gains, while fully expensing purchased and internal investment. Under current tax bases, eliminating capital-gains taxation yields the lowest deadweight loss among a range of simple reforms, by increasing entry and capital reallocation.

KEY WORDS: business transfers, capital allocation, firm dynamics, capital taxation

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1 Introduction

Private businesses are central to the U.S. economy, generating over half of all business income and close to half of the private-sector wage bill. When these businesses are sold, most of the value transferred consists of intangible assets such as customer bases, trade names, and goodwill. This capital cannot easily be rented or traded in centralized markets and is typically transferred indivisibly and valued only when a business is sold. Yet quantitative theories of firm dynamics and capital allocation, as well as empirical approaches to measuring capital and estimating production and investment technologies, have largely abstracted from these assets because the data needed to discipline them are scarce. Neither the stocks nor the expenditures that create them are directly observed. Moreover, capital value is revealed only through infrequent private-business transfers, which are not publicly disclosed. This omission leaves open basic questions about how private firms grow, how capital is allocated and valued, and how business capital stocks, exchanges or returns should be taxed. We update the standard theory of firm dynamics to incorporate essential properties of private-business capital. We discipline the theory using novel administrative data from the Internal Revenue Service (IRS) on private-business income, expenditures, and transfers, and use the model to revisit the basic principles for taxing business income and capital gains.

Our environment builds on the neoclassical models of firm dynamics in Lucas (1978) and Hopenhayn (1992), but distinguishes productive inputs by how they can be reallocated. Owner productivity evolves stochastically and is inalienable. Business capital can be transferred but not rented. External factors, including employee time and generic fixed assets such as office space and equipment, are rented in spot markets. Owners accumulate business capital either by investing internally or by purchasing another business. Internal investment requires costly owner effort. Business sales take time to complete, occur through bilateral meetings, and transfer the seller's entire stock of business capital. After a sale, the seller can start a new business or become an employee. These assumptions capture three salient features of private-business capital reallocation: time to trade, bilateral exchange, and indivisibility.

The trading protocol gradually reallocates capital toward owners who value it most. In the calibrated equilibrium, owners with relatively low marginal products of capital sell to owners with relatively high marginal products. We characterize this reallocation as an optimal assignment and show that a stationary competitive equilibrium is efficient. On executed sales, the transaction price depends only on the quantity of capital sold. In the calibrated equilibrium, per-unit prices are lower for sufficiently large transfers. Because trade takes time and involves indivisibilities, even the efficient allocation exhibits dispersion in marginal products of capital and returns on business wealth. These predictions are central to our quantitative analysis: trading patterns and transaction prices discipline the production and investment technologies for otherwise unobserved capital, while the resulting dispersion in marginal products and returns informs our

measurement of business wealth and our analysis of tax policy.

We calibrate the model using U.S. administrative tax records for S corporations, the most prevalent type of corporation in the United States. S corporations are pass-through entities whose ownership restrictions allow us to link businesses to their individual owners. By linking business and individual tax forms, we construct longitudinal panels spanning business and owner life cycles. While U.S. Census microdata have been used in the firm dynamics and capital allocation literature, the focus has been on manufacturing firms and their property, plant, and equipment. For the capital analyzed here, which is relevant for most business activity in the economy, there is no analogue of the Census microdata on investment expenditures, adjustment costs, and capital retirements used in previous studies.

A distinctive feature of the IRS data is that we observe business sales, including the purchase price, the identities of the buyer and seller, and the allocation of the purchase price across marketable securities, fixed assets, and intangible assets.[1] Intangible assets constitute roughly 70 percent of the transferred value in a typical sale. This share is likely a lower bound on the importance of business capital with the properties we model. Assets conventionally classified as fixed assets, such as customized equipment, can also be difficult to divide, rent, or redeploy outside the business. Taken together, these IRS transactions provide useful moments when calibrating our model of the capital market: who trades with whom and the terms of the trade. We map the model predictions to their empirical counterparts in the transaction data and use them to identify the output elasticity and investment-cost parameters for private business capital.

We examine the implications of our model of private firm dynamics for the allocation of capital across firms. As noted above, our competitive equilibrium is efficient and hence features no misallocation. However, the model predicts substantial dispersion in marginal products of capital due to the nature of business technologies and transfers. In our baseline, the standard deviation of the log of the marginal product of capital is 57 percent. We explore and quantify the roles of time to trade and indivisibility in capital exchange by comparing our baseline estimate of marginal product of capital dispersion to alternatives across a range of trade frequencies and assumptions about capital divisibility. We find significant dispersion across reasonable variations of those two features.

We also use the model to characterize business-capital adjustment over the firm life cycle. Although neither business-capital stocks nor internal investment expenditures are directly observed, we construct investment rates from a simulated panel using the same accounting conventions as studies of physical capital. The model generates small positive adjustments through internal in-

[1]The data come from Form 8594, which must be filed by both buyers and sellers. Fixed assets include inventory, equipment, buildings, and land. Intangible assets include customer- and information-based intangibles, non-compete covenants, licenses and permits, franchises, trademarks, trade names, workforce in place, business books and records, processes, designs, and patterns, as well as goodwill and going-concern value. These allocations determine capital gains for sellers and the tax bases for depreciation and amortization by buyers.

vestment and large adjustments through acquisitions and whole-business sales. While the modal firm-year investment rate shows a modest adjustment, most investment is concentrated, with frequent spikes in rates over 20 percent. The annual frequency of business sales is 4.8 percent. Overall, the adjustment behavior, especially for more mature firms, is comparable to the U.S. Census evidence from capital expenditures in manufacturing plants and provides an additional check on the model's capital accumulation and reallocation mechanisms.

The model also makes predictions for the stock of business wealth and its return. We explore two familiar but distinct measures of wealth. The first is the value of transferable wealth, often reported in surveys such as the Survey of Consumer Finances and particularly relevant for analyzing the taxation of capital gains. The second is the total value of the going concern, which is the present discounted value of payments to the current owner. This measure includes not only the business capital that can be transferred, but also the owner's productivity and future growth prospects. We estimate an aggregate total value of 2.59 times private sector value added, with 18.5 percent of value that is transferable. Finally, we document significant dispersion across owners in both the ratio of transferable to total wealth and in returns to business wealth. This dispersion cautions against making simple imputations of wealth inequality based on value-to-book or value-to-income multiples for publicly listed corporations.

Our analysis of capital trade, valuations, income yields, and marginal products provides the objects needed to study business taxation. We first characterize an undistorted benchmark. In a special case with rentable fixed assets but no business capital, the environment reduces to the standard neoclassical user-cost framework of Hall and Jorgenson (1967). When capital can be rented and divisibly adjusted, a uniform cash-flow tax leaves productive decisions undistorted if investment is fully expensed and net cash flows from different productive activities are taxed at the same rate. The same principle continues to apply when capital is self-created through a convex technology and reallocated through infrequent, indivisible trades, but its implementation becomes more demanding. Full expensing must cover both purchased capital and the owner's effort cost of internal investment. Realized gains from selling self-created capital must also be taxed at the same rate as other business and labor income. Because owner effort is not observable to the tax authority, no feasible tax system implements this deduction, and every system we study distorts internal investment.

Against this benchmark, we quantify the outcomes of the U.S. tax system and several revenue-neutral reforms. The U.S. system in our model generates a deadweight loss in private consumption of 5.2 cents per dollar of revenue. Purchased intangible assets are amortized rather than immediately expensed, while much of the imputed cost of creating intangible capital internally cannot be deducted. In the U.S. system, realized capital gains are also taxed at a lower rate than labor and business income. Relative to the ideal system, the average capital stock and the fraction of capital traded are 6.8 percent and 47.1 percent lower, respectively. We consider two reforms that

retain existing U.S. tax bases and expensing rules: repealing the capital gains tax and replacing the lost revenue with income taxation, and eliminating the preferential treatment of capital gains by equalizing the tax rates. Because neither reform provides full expensing, equalizing the rates intensifies the trade distortion without correcting the investment distortion. Repealing the capital gains tax therefore produces better outcomes, primarily by improving capital reallocation and entry. Finally, replacing the capital gains tax with an annual tax on imputed transferable wealth avoids the realization-based lock-in effect but discourages business entry. Compared with income taxation, the wealth tax falls proportionally more on less productive owners whose entry and exit choices are more elastic.

1.1 Related Literature

Our paper relates to the extensive body of work studying entrepreneurship, firm dynamics, productivity, and the allocation, valuation, and taxation of business capital.

The model we develop provides a bridge between the entrepreneurship literature starting with Lucas (1978) and the firm dynamics literature starting with Jovanovic (1982) and Hopenhayn (1992). On the theoretical side, we retain much of the neoclassical spirit of these earlier frameworks but introduce technological and market-specific features relevant for most business capital currently used in production. In particular, we model capital assets as indivisible and non-rentable and the sale of a business as a transfer of a group of assets. In considering the indivisible nature of the trade, we are also building on Holmes and Schmitz (1990), who focus on heterogeneity in the ability to start firms and on empirical measures of serial entrepreneurship.

Our work is connected to the literature on mergers, acquisitions, and the sale of business capital that uses models of random search with bargaining or directed search with one-sided heterogeneity to ensure tractability. See, most notably, Jovanovic and Rousseau (2002) and David (2021) on mergers and acquisitions, Gavazza (2016) and Ottonello (forthcoming) on fixed asset reallocation, and more recently, Gaillard and Kankanamge (2020) and Guntin and Kochen (2024) on the role of financial frictions in buying and selling firms. Unlike this literature, we model business sales as transactions in competitive, decentralized markets that are subject to time-to-trade, indivisibility, and a fixed resource cost. Using tools from the matching literature (Galichon et al. (2019)), we solve for the equilibrium set of prices and demonstrate that it leads to an efficient allocation. We find this efficiency property appealing as it allows us to isolate the dispersion in marginal products generated solely by the indivisibility and infrequent trade of capital in private businesses. Furthermore, our focus is different: we use our model to generate predictions for the distribution of marginal products of capital, valuations, and returns for private businesses and derive implications for tax policy.

Our paper is also related to the literature on capital measurement and misallocation. Much of

this literature has focused on the role of regulatory, financial, and informational constraints and capital adjustment costs. See, among others, Ramey and Shapiro (2001), Hsieh and Klenow (2009), Asker et al. (2014), Restuccia and Rogerson (2017), David and Venkateswaran (2019), Sterk et al. (2021), and Jaimovich et al. (2025); Cooper and Haltiwanger (2006), Baley and Blanco (2021), and Lippi and Oskolkov (2023) focus specifically on capital indivisibility. Differently from this literature, we study non-rentable intangible business capital, which is the dominant form of capital owned by U.S. private firms, rather than physical capital in manufacturing or public firms. Because investment in and stocks of these assets are not directly observed, the standard approaches used to estimate output elasticities and investment costs cannot be applied. We instead exploit IRS data on business sales combined with model-guided identification and use the resulting estimates to generate predictions for the distribution of marginal products, valuations, and returns for private businesses.

Estimates of business value for traded and non-traded private firms have been inputs to the growing literature on measuring private business wealth. Most studies in this area rely on structural models of entrepreneurship guided by survey data (see Cagetti and De Nardi (2006)) or estimation methods to capitalize income and investment flows (see Saez and Zucman (2016), Crouzet and Eberly (2023), Smith et al. (2023), Sveikauskas et al. (2024), Gomez and Gouin-Bonenfant (2025), and He et al. (2024)). Our approach to measuring business value instead uses model-implied valuation concepts, informed by detailed data on business sales and the income statements of the buyers and sellers of businesses.

Finally, one of our main contributions is to the public finance literature on the taxation of stocks, flows, and returns on capital. In standard models with perfect capital markets, the normal return to capital should go untaxed (Atkinson and Stiglitz 1976), and for businesses this prescription is implemented by taxing cash flows uniformly with investment fully expensed, in order to preserve production efficiency (Diamond and Mirrlees 1971; Hall and Jorgenson 1967). We develop the counterpart of this benchmark for self-created business capital and use it to evaluate the U.S. tax system and several proposed alternatives. Our analysis highlights the role of taxes on realized gains in the lock-in of capital with current owners—the effect studied by Chari et al. (2003) and Cavalcanti and Erosa (2007) in the Holmes and Schmitz (1990) framework—and in entry. The same margins govern our evaluation of proposed policies that are intended to tax gains before they are realized, as in Azevedo et al. (2025), who study startup founders, and Guvenen et al. (2023), who make an efficiency case for wealth taxation when financial frictions generate dispersion in returns.[2] In our model, dispersion in business returns arises from the indivisibility and infrequent trade of capital; a wealth tax leaves this source untouched and operates instead by dis-

[2]In recent work, Aguiar et al. (2025) also reexamine the optimality of taxing financial wealth and capital gains, particularly when revaluations interact with redistribution and insurance considerations. These motives are absent from our analysis, which focuses on productive business capital.

couraging the creation of business wealth. Throughout, we discipline the trading responses that drive the policy rankings with the empirical evidence on capital-gains realization elasticities—for example, Bakija and Gentry (2014) and Agersnap and Zidar (2021).

The rest of the paper is organized as follows. Section 2 documents the composition of capital in U.S. private businesses, which motivates the theory that follows. Section 3 details the environment, including timing of events, descriptions of problems solved by business owners, and a definition of a recursive equilibrium. A characterization of the equilibrium is provided in Section 4. In Section 5, we document statistical properties of U.S. firm-level data that guide the calibration described in Section 6. In Section 7, we document the model's predictions for the dispersion of marginal products of capital, the margins of capital adjustment, and business wealth. In Section 8, we assess the impacts of business taxation. Section 9 concludes.

2 Business Capital in U.S. Private Firms

The literature on investment, adjustment costs, and misallocation has largely centered on physical capital in U.S. firms. This capital is relatively easy to measure using aggregate data from the fixed asset tables produced by the Bureau of Economic Analysis (BEA), capital expenditures from the Annual Survey of Manufactures, and annual 10-K filings for listed firms from the Securities and Exchange Commission. In this section, we document two facts that motivate our theory: the physical capital owned by private businesses is small relative to their output, and, relatedly, most of the value transferred when private businesses are sold is intangible.

Private businesses are predominantly organized as pass-through entities (S corporations, partnerships, and sole proprietorships) whose income is taxed at the owner level.[3] Of these, S corporations and partnerships file both income statements and balance sheets with their tax returns, thus providing a direct measure of the physical capital-output ratio for private businesses. Specifically, we use aggregate data published by the IRS's Statistics of Income (SOI) program and compute the net book value of depreciable assets plus inventories relative to a measure of output given by business receipts less cost of goods sold. In tax year 2022, this book-value capital-output ratio was 0.56 for S corporations and partnerships (see U.S. Internal Revenue Service 2022).[4] If we revalue the historical-cost stocks reported on IRS balance sheets using BEA price indices and depreciation rates, we find a current-cost capital-output ratio of 0.81.[5] To put these numbers in perspec-

[3]For more details, see Bhandari et al. (2026), which analyzes patterns of income growth and determinants of entrepreneurial choice for pass-through business owners using IRS data.

[4]We exclude partnerships in real estate, finance, and holding companies, whose receipts largely accrue outside of business receipts (for example, the rental income of real estate partnerships is reported separately), so that a comparable measure of output cannot be constructed for them. Excluding S corporations in these industries as well leaves the estimate essentially unchanged.

[5]Specifically, we scale the net depreciable assets reported on tax returns by the ratio of the current-cost to the historical-cost net stock of equipment and structures for nonfinancial corporations, which is 1.61 for 2022. (See Fixed

tive, Cooley and Prescott (1995) estimate an economy-wide capital-output ratio of 3.3, the value typically cited in the macro literature. We conclude that the physical capital owned by private businesses, while easy to measure, is very small.[6]

Measuring the intangible capital of private firms is harder: investments in customer bases, trade names, and goodwill are expensed rather than capitalized, and the stock of these assets is observed only when a business changes hands. Purchase price allocations from business sales provide a direct window. Using brokered sales from the Pratt's Stats database, Bhandari and McGrattan (2021) document that intangible assets account for roughly 70 percent of the value transferred in a typical sale of a private business. In Section 5, we corroborate this estimate using administrative data from e-filed IRS forms on purchase price allocations from counterparty tax filings. We conclude that the intangible capital owned by private businesses, while hard to measure, is very large.

Although the labels "tangible" and "intangible" are compelling from a measurement perspective, some assets owned by private firms escape this sharp distinction. Consider, for example, a specialized restaurant kitchen, a vehicle lift in an auto repair shop, or an elevator system in a hotel. They are conventionally classified as tangible assets, but, just as customer relationships and trade names, they are customized to the business, cannot easily be rented or reallocated, and are often transferred and valued only as part of the going concern. The 70 percent share of transferred value classified as intangible arguably constitutes a lower bound on the share of capital with such properties in those transactions. Taken together, these facts suggest that most of the capital owned by private businesses is firm-specific, is rarely observed unless the business is sold, and is not rented or traded in centralized markets. The theory and measurement we develop next place this type of capital at the center of the analysis of private-firm dynamics.

3 Theory

The economy is populated by a unit measure of individuals who choose between running a business and working in paid employment, and who may switch occupations over time. Business owners produce a homogeneous consumption good using three types of inputs that differ in divisibility and transferability. The first is entrepreneurial skill, which evolves stochastically and cannot be transferred. The second is intangible business capital, which is built through costly investment and can be sold, but only in its entirety and only when a trading opportunity arrives. The third consists of external factors (labor and fixed assets such as equipment and office space) that are rented in competitive spot markets. Business capital is the only input that owners accumu-

Asset Tables 4.1 and 4.3 published by the BEA.)

[6]Our empirical analysis and calibration later focus on S corporations as the main sample. They are the most prevalent type of corporation in the United States, and their owners must be individuals. In Section 5, we report the corresponding capital-output ratio for S corporations.

late and trade, and we refer to it simply as *capital* in what follows. We always refer to the rentable capital input as fixed assets. We describe the environment, the owner's dynamic program, and a stationary recursive equilibrium in turn.

3.1 Environment

Production. Let $x = (z, k) \in \mathcal{X} = Z \times K$ denote the owner's state, where $Z \subseteq \mathbb{R}$ is the set of productivities and $K \subseteq \mathbb{R}_+$ is the set of capital stocks, and use $z(x)$ and $k(x)$ to denote the components of x. Output is produced using the technology $y(x, b, n)$, where b denotes fixed assets and n denotes labor. The technology is strictly increasing in productivity and inputs, strictly concave in (b, n), and exhibits decreasing returns to scale in the inputs (k, b, n).

Productivity process. Productivity follows a diffusion

$$\mathrm{d}z = \mu_z(z)\,\mathrm{d}t + \sigma_z(z)\,\mathrm{d}\mathcal{W}_z,$$

where $\mu_z(\cdot)$ is the drift, $\sigma_z(\cdot)$ is the volatility, and $\mathcal{W}_z$ is a standard Wiener process. Productivity evolves only while the individual is a business owner, and it is instead fixed while working.

Preferences. Owners and workers are risk-neutral and discount the future at rate ρ. Investment in business capital requires the owner's effort: producing new capital at rate i generates disutility $c(i)$, measured in units of the consumption good, where c is strictly increasing and strictly convex with $c(0) = 0$. This disutility of effort enters preferences additively in consumption-equivalent units.

Capital accumulation. Capital evolves according to

$$\mathrm{d}k = (i - \delta_k k)\mathrm{d}t + \sigma_k k\,\mathrm{d}\mathcal{W}_k,$$

where $i \geq 0$ is gross investment, δ_k is the depreciation rate, σ_k governs the volatility of capital innovations, and $\mathcal{W}_k$ is a standard Wiener process. We interpret the diffusion term as a shock to firm-specific capital rather than to the owner's productivity. For example, it may capture shocks to the firm's customer base, as when customers randomly encounter the store, or shocks to its supply chain, as when an upstream firm exits.

Capital is also subject to an occasional large depreciation shock. At Poisson rate δ_{exo}, the entire stock is destroyed. When this shock arrives, the owner loses her capital but retains productivity z. We interpret this event as firm exit, with the owner retaining the option to start a new business or switch occupations.

Rental markets. The rental markets for labor, n, and fixed assets, b, are perfectly competitive with unit costs w and r, respectively. We assume that the fixed assets are owned by a competitive mutual fund sector that can convert consumption goods to investment one-for-one and faces a depreciation rate of δ_b.

Capital market. The market for capital departs from the neoclassical framework in three dimensions: trading opportunities are intermittent, transactions are bilateral, and capital is indivisible. An owner with state x accesses the capital market at Poisson rate η. We refer to this intermittent access as *time to trade*. Completing a transaction requires a resource cost $c_T \geq 0$, which we assume is paid by the buyer.[7] We think of this as the fee charged in intermediating the sale.

When an owner enters the market, she faces contracts indexed by the state of the counterparty and by her side of the transaction. Let $\tilde{x}$ denote the counterparty's state. The index j records whether the owner with state x is buying from the counterparty, $j = b$, or selling to the counterparty, $j = s$. For each $\tilde{x} \in \mathcal{X}$ and $j \in \{b, s\}$, the contract specifies a price $p^j(x, \tilde{x})$ and a capital choice $k^j(x, \tilde{x})$. This role index is needed because the same pair of states $(x, \tilde{x})$ can correspond to either side of a bilateral transaction.

Consider an owner with state x deciding on a trade with a counterparty that has state $\tilde{x}$. We refer to the ability to trade with only one counterparty at a time as *bilateralness*. As a buyer, the owner with state x pays $p^b(x, \tilde{x})$ to the counterparty and exits the trading stage holding capital $k^b(x, \tilde{x})$. As a seller, this owner receives $p^s(x, \tilde{x})$ and exits with capital $k^s(x, \tilde{x})$. Since there is no difference between an owner with state x buying from an owner with state $\tilde{x}$ and an owner with state $\tilde{x}$ selling to an owner with state x, the prices of the two transactions must be equal: $p^b(x, \tilde{x}) = p^s(\tilde{x}, x)$. The functions $k^j : \mathcal{X}^2 \to K$ and $p^j : \mathcal{X}^2 \to \mathbb{R}$, for $j \in \{b, s\}$, are determined as part of an equilibrium that we define later.

For the allocation of capital within a match, we impose that an owner can either buy the entire capital stock of the counterparty or sell their entire capital stock. This amounts to the following restriction on k^j: for all pairs $(x, \tilde{x}) \in \mathcal{X}^2$,

$$k^b(x, \tilde{x}) = k(x) + k(\tilde{x}) \text{ and } k^s(x, \tilde{x}) = 0. \tag{1}$$

We refer to restriction (1) as *indivisibility*. This captures a key feature of our model, namely, that the reallocation of capital across owners in bilateral trades occurs in a "lumpy" fashion.

Owner entry, exit, and occupational choice. Entry into and exit from the economy occur at Poisson rate ψ_e. The entrant distribution over states $x = (z, k)$ is denoted by $G = G_z \otimes \delta_0$, where δ_0 is the unit mass at $k = 0$: newborns start with no business capital and draw productivity $z \sim G_z$.

[7] In our baseline setup, assigning the transaction cost to the buyer is without loss of generality: we could equivalently assign it to the seller or divide it between the two, with the allocation of capital invariant and prices adjusting in equilibrium. Later in section 8, when we introduce capital gains taxes, the incidence is no longer a normalization.

Given the draw, they decide whether to become workers or business owners, with the option of switching occupations at Poisson rate ψ_o. Workers who have productivity z compare the value of continuing work and the value of switching to business ownership.

Discussion of assumptions. Before proceeding, we discuss some of our simplifying assumptions. We impose them intentionally to highlight the novel aspects of business capital accumulation and trade. Each of these assumptions could be relaxed within our framework, at the cost of additional data requirements of varying stringency.

First, we assume that buyers' and sellers' capital stocks combine additively. In practice, mergers may generate synergies or diseconomies of scale that affect valuations and trade decisions. Since we only observe the price paid for capital when a trade occurs, our data cannot distinguish whether a higher (lower) price reflects more (less) capital or instead the presence of synergies (diseconomies). Incorporating post-trade income gains of buyers could help address this limitation and allow for richer forms of capital aggregation.

Second, workers earn a constant wage w and do not accumulate human capital. This simplification keeps the analysis focused on business ownership and capital reallocation. In richer environments, workers accumulate skills with experience, which could affect their productivity as future business owners. Worker heterogeneity would then influence the response of entry to business taxation.

Third, we assume that the owner's effort is the sole input into the accumulation of business capital. While effort is undoubtedly central, other inputs such as hired labor and materials may also contribute to building firm value. Relaxing this assumption to allow substitution across inputs is straightforward, though empirically challenging because it requires distinguishing investment expenditures from current production costs.[8]

Finally, we adopt linear preferences as a point of departure. This assumption isolates the roles of trading frictions and indivisibilities in shaping capital allocation and efficiency. The framework can be extended to incorporate the economic consequences of imperfect financial markets, such as undiversifiable risk and borrowing constraints.

3.2 Recursive Formulation

In this section, we describe the dynamic programs of owners and workers and the law of motion for the distribution of owners over states induced by the policy functions.

Owner and Worker value. Let $V : \mathcal{X} \to \mathbb{R}_+$ denote the value of an owner. An owner who accesses the capital market chooses a trading strategy $\lambda = (\lambda^b, \lambda^s)$: for each state x, $\lambda^j(x, B)$ is the

[8] See Bhandari and McGrattan (2021) for a related discussion and measurement of other inputs.

probability of trading as the buyer ($j = b$) or the seller ($j = s$) with a counterparty whose state lies in the set $B \subseteq \mathcal{X}$, with $\sum_j \lambda^j(x, \mathcal{X}) \leq 1$. We write $\lambda^j(x, \tilde{x})$ for the density of this kernel (in the finite-state case, simply the probability of trading in role j with type $\tilde{x}$), so that integrals such as $\int \lambda^j(x, \tilde{x})\, d\tilde{x}$ denote its total mass. This formulation lets us model choices over whom to trade with and whether to buy or sell.

Let $V_{trade}(\,\cdot\,; \lambda) : \mathcal{X} \to \mathbb{R}$ be the owner's gains from trade for a given λ. Let $W : Z \to \mathbb{R}_+$ be the value of being a worker. Given the contracts $\{p^j, k^j\}_{j \in \{b,s\}}$, the owner value solves the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}(\rho + \psi_e)V(x) = \max_{n,b,i,\lambda}\ & y(x,b,n) - rb - wn + \partial_k V(x)(i - \delta_k k) - c(i) + \frac{1}{2}\sigma_k^2 k^2 \partial_{kk} V(x) \\ & + \delta_{\text{exo}}\left(V(z,0) - V(x)\right) + \mu_z(z)\partial_z V(x) + \frac{1}{2}\sigma_z(z)^2 \partial_{zz} V(x) \\ & + \psi_o\{W(z) - V(x)\}^+ + \eta V_{trade}(x; \lambda),\end{aligned} \tag{2}$$

The term on the left-hand side is the flow value of being an owner with state x. The right-hand side includes the operating profit after paying the rental cost of fixed assets and the wage bill; the gain from capital investment net of the cost of investment; the changes in value induced by the capital shocks—the diffusion in σ_k and the occasional destruction of the entire stock at rate δ_{exo}; the changes in value induced by the evolution of productivity z; the option value of exiting self-employment; and the expected gains from trade from accessing the market for capital, V_{trade}.

The last term of the HJB equation (2) is absent from traditional firm dynamics models. Given the contracts $\{p^j(x, \tilde{x}), k^j(x, \tilde{x})\}_{\tilde{x} \in \mathcal{X},\, j \in \{b,s\}}$, define $v^j(x, \tilde{x})$ as the value for an owner with state x who trades with a counterparty with state $\tilde{x}$ in role j:

$$\begin{aligned} v^b(x, \tilde{x}) &\equiv V(z(x), k^b(x, \tilde{x})) - p^b(x, \tilde{x}) - c_T \\ v^s(x, \tilde{x}) &\equiv V(z(x), k^s(x, \tilde{x})) + p^s(x, \tilde{x}). \end{aligned}$$

As a buyer, the owner pays p^b and the transaction cost c_T to acquire the counterparty's capital, while as a seller the owner receives p^s and parts with its own capital. The gains from trade are given by

$$V_{trade}(x) = \max_{\lambda:\, \sum_{j \in \{b,s\}} \int \lambda^j(x,\tilde{x})\, d\tilde{x} \leq 1} V_{trade}(x; \lambda), \tag{3}$$

where

$$V_{trade}(x; \lambda) \equiv \sum_{j \in \{b,s\}} \int \{v^j(x, \tilde{x}) - V(x)\} \lambda^j(x, \tilde{x})\, d\tilde{x}.$$

The constraint holds with slack whenever the owner does not trade with positive probability, and the residual mass $1 - \sum_{j \in \{b,s\}} \int \lambda^j(x, \tilde{x})\, d\tilde{x}$ is the probability of leaving the market without trading. Because leaving the market without trading is always feasible, the maximized gains satisfy $V_{trade}(x) \geq 0$.

Since the productivity z of an individual changes only while actively running a business, a

worker's state and payoff do not change over time. Once an individual decides to be a worker, re-entry into business ownership therefore never becomes strictly profitable, and the choice is never overturned. The value of being a worker, W, is the present value of wages until the exogenous stochastic time of death and given by

$$W = \frac{w}{\rho + \psi_e}. \tag{4}$$

The entry and exit decisions into business ownership are denoted by

$$\iota^{\text{in}}(x) = \mathbf{1}\{V(x) > W\}, \tag{5}$$

$$\iota^{\text{out}}(x) = \mathbf{1}\{W > V(x)\}, \tag{6}$$

where $\mathbf{1}\{\cdot\}$ is the indicator function. The strict inequalities break ties against switching: an individual indifferent between the two occupations, $V(x) = W$, neither enters nor exits business ownership.

Law of motion for the distribution of owners. Let $\phi \in \mathcal{M}_+(\mathcal{X})$ be the measure over owner states, where $\mathcal{M}_+(\mathcal{X})$ denotes the set of finite nonnegative measures on $\mathcal{X}$. The total mass of ϕ is the endogenous fraction of owners in the economy. We write $\int f \, d\phi$ for integration against ϕ and, where unambiguous, use the shorthand $\int f(x)\phi(x) \, dx$.

Let $\mathcal{A}_{(i,\lambda)}$ denote the infinitesimal generator of the owner's state implied by the Bellman equation (2): acting on a test function h, it collects the drift $i - \delta_k k$ in capital, the diffusion terms in z and k, the trade jumps at rate $\eta\lambda^j$, the destruction jump to $(z, 0)$ at rate δ_{exo}, and the removal of owners through death at rate ψ_e and occupational exit at rate $\psi_o \iota^{\text{out}}$. For every test function h in the domain of the generator, the time evolution of the ϕ-weighted average of h is given by

$$\frac{d}{dt}\int h \, d\phi = \int \mathcal{A}_{(i,\lambda)} h \, d\phi + \psi_e \int h(x) \, \iota^{\text{in}}(x) \, dG(x). \tag{7}$$

The first term on the right hand side of equation (7) describes the evolution of the distribution of owners due to investment, productivity and capital shocks, trades, destruction, and exit. The second term describes the entry of new owners into the economy. We describe the evolution of ϕ in weak form to allow for point masses. Equivalently, the law of motion of the measure ϕ can be expressed as $\dot{\phi} = \mathcal{A}^*_{(i,\lambda)}\phi + \psi_e \, \iota^{\text{in}} G$ in the sense of distributions, where $\mathcal{A}^*_{(i,\lambda)}$ is the adjoint of the generator.

Furthermore, we can compute the implied mass of business owners, $m = \int d\phi$, by setting $h \equiv 1$ in (7): trades reallocate capital across owners without changing their number, so only entry, death, and occupational exit contribute to changing the mass of owners. The mass evolves according to:

$$\dot{m} = \psi_e \int \iota^{\text{in}}(x) \, dG(x) - \psi_e m - \psi_o \int \iota^{\text{out}}(x) \, d\phi(x). \tag{8}$$

The first term is the entry flow of newborns who choose business ownership. The second term is the outflow due to the death of current owners. The third term is the outflow of owners who

receive an occupational-switch opportunity and move into the labor market.

We are now ready to define an equilibrium.

3.3 Equilibrium

Definition 1. A *stationary recursive equilibrium* is given by (*i*) value functions $V : \mathcal{X} \to \mathbb{R}_+$ and $W \in \mathbb{R}_+$; (*ii*) policy functions $n : \mathcal{X} \to \mathbb{R}_+$, $b : \mathcal{X} \to \mathbb{R}_+$, $i : \mathcal{X} \to \mathbb{R}_+$, $\lambda^j : \mathcal{X}^2 \to \mathbb{R}_+$, $\iota^{\text{in}} : \mathcal{X} \to \{0,1\}$, $\iota^{\text{out}} : \mathcal{X} \to \{0,1\}$; (*iii*) wage, w and rental rate, r; (*iv*) capital contracts $p^j : \mathcal{X}^2 \to \mathbb{R}$ and $k^j : \mathcal{X}^2 \to K$, with $j \in \{b,s\}$; (*v*) a measure over owner states $\phi \in \mathcal{M}_+(\mathcal{X})$ such that:

- Given $\{k^j(\cdot), p^j(\cdot)\}_{j\in\{b,s\}}$ and wage w, the value function for owners and workers $\{V(\cdot), W\}$ solve the Bellman equations (2) and (4).
- The policy functions for investment, trade, and rentable inputs solve the maximization problem in equation (2). The entry and exit decisions for an owner satisfy (5) and (6).
- The rental rate for fixed assets satisfies $r = \delta_b + \rho$ and the labor market clears:
$$\int (1 + n(x))\, \phi(x) dx = 1. \tag{9}$$
- The trading arrangements are feasible and consistent. That is, for all pairs $(x, \tilde{x}) \in \mathcal{X}^2$, the capital allocations satisfy the indivisibility restriction (1), the prices satisfy the consistency requirement
$$p^b(x, \tilde{x}) = p^s(\tilde{x}, x), \tag{10}$$
and the trading policies satisfy the consistency condition
$$\lambda^b(x, \tilde{x})\phi(x) = \lambda^s(\tilde{x}, x)\phi(\tilde{x}). \tag{11}$$
That is, the mass of owners with state x who buy from those with state $\tilde{x}$ equals the mass of owners with state $\tilde{x}$ who sell to those with state x.
- The measure over owner types is stationary: $\dot{\phi} = 0$.

4 Characterizing the Equilibrium

In this section, we provide a characterization of the equilibrium and discuss its properties. We compute the equilibrium in two steps. First, we take the value function V and the equilibrium measure of firms ϕ as given and characterize the prices p^j, the allocations—that is, the trading rates λ^j and the post-trade capital stocks k^j—and the gains from trade V_{trade} that are consistent with market clearing. Second, we solve for optimal (V, ϕ) given the menu of prices and terms of trades. All proofs are in the appendix.

4.1 Characterizing prices and allocations given (ϕ, V)

As a first step, we show that the equilibrium prices and allocation of capital can be characterized by an *assignment* problem that maximizes total surplus—which is measured using V—by assigning capital in bilateral trades, given the measure of owner types in the economy ϕ.

A simple example. Before formalizing this problem, we consider a static example in order to provide some intuition about who trades with whom and how the terms of trade are determined. For the example, we use a simple production function, namely, $y = zk$, set the transaction cost c_T to zero, and assume that there are 20 owners with $z = 1$, 10 owners with $z = 0$, and all 30 have $k = 1$. Consider an allocation that is achieved by the low-z types selling their businesses to any 10 of the high-z types, with trading rates equal to $\lambda^s(x_L, x_H) = 1$ and $\lambda^b(x_H, x_L) = 1/2$ and the prices equal to $p^b(x_H, x_L) = p^s(x_L, x_H) = 1$. It is easy to check that these allocations and prices implement trading arrangements that satisfy feasibility, consistency, and private optimality. A complete equilibrium also specifies the contracts for trades not taken on the equilibrium path. All remaining trading rates are zero (that is, $\lambda^b(x_L, x_H) = \lambda^s(x_H, x_L) = 0$, and no same-type trades occur), and the off-path prices can be chosen to deter the trades not taken, leaving no profitable deviation. The natural choice is a single price of capital: set $p^b(x, \tilde{x}) = p^s(\tilde{x}, x) = 1$ for every pair, the price at which capital trades on the equilibrium path.

We make a few observations about the trading outcome. First, the allocation of capital maximizes total surplus across all possible pairs of owners. Second, the terms of the trade are such that the surplus split is determined by the short side of the market. In particular, the prices are such that the high-z types are indifferent between trading and not trading. We now extend these observations to the general case.

Trades as an assignment problem. Let the surplus from matching for a pair $(x, \tilde{x})$ be defined as follows:

$$S(x, \tilde{x}) = \max \left\{ V(z, k + \tilde{k}) + V(\tilde{z}, 0),\ V(z, 0) + V(\tilde{z}, k + \tilde{k}) \right\} - (V(x) + V(\tilde{x})) - c_T.$$

The two arguments are the possible trades in a match, namely, the owner with state x buys capital from or sells capital to the owner with state $\tilde{x}$, respectively. In either case, the pair incurs the transaction cost c_T, which is therefore netted from the surplus. The outcome that attains the maximum pins down the post-trade capital stock k^j. In other words, when the owner with state x buys, we have $k^b(x, \tilde{x}) = k(x) + k(\tilde{x})$ and $k^s(\tilde{x}, x) = 0$, and symmetrically when owner x sells.

When setting up the assignment problem, we split the measure ϕ into two measures ϕ^a and ϕ^b such that for $x \in \mathcal{X}$ we have

$$\phi^a(x) = \phi^b(x) = \frac{\phi(x)}{2}.$$

Given $\{\phi^a, \phi^b\}$, an *assignment* is a measure $\gamma \in \mathcal{M}_+(\mathcal{X}^2)$ over pairs of states, written via its density $\gamma(x, \tilde{x})$: the mass of matches between an owner with state x drawn from ϕ^a and a counterparty with state $\tilde{x}$ drawn from ϕ^b. The surplus-maximizing assignment solves

$$Q(\phi, V) = \max_{\gamma \geq 0} \int S(x, \tilde{x}) \gamma(x, \tilde{x}) \, dx \, d\tilde{x} \tag{12}$$

such that for $x \in \mathcal{X}$

$$\int \gamma(x, \tilde{x}) d\tilde{x} \leq \phi^a(x) \tag{13}$$

$$\int \gamma(\tilde{x}, x) d\tilde{x} \leq \phi^b(x). \tag{14}$$

We label this problem as *P1*. The slack in (13) and (14) corresponds to owners who leave the market unmatched. Attach multipliers $\mu^a(x)$ and $\mu^b(x)$ to constraints (13) and (14). An optimal pair (μ^a, μ^b) is a *dual solution* of *P1*, and a dual solution is *symmetric* if $\mu^a = \mu^b$.

The next theorem shows that problem *P1* characterizes the trading block of an equilibrium in both directions: the solution of *P1* delivers prices, capital allocations, and trading rates with the properties required by Definition 1, and, conversely, the trading arrangements of *any* equilibrium solve *P1*. To state both directions at a fixed pair (ϕ, V), call $(p^j, k^j, \lambda^j)_{j \in \{b,s\}}$ *admissible trading arrangements given* (ϕ, V) if the capital allocations satisfy the indivisibility restriction (1), the prices satisfy the consistency requirement (10), the trading rates satisfy the consistency condition (11) and the probability constraint in (3), and $\lambda(x, \cdot)$ attains the gains from trade $V_{trade}(x)$ in (3) at every $x \in \mathcal{X}$. These are exactly the conditions that Definition 1 imposes on the trading block of an equilibrium.

Theorem 1. *Fix (ϕ, V). (i) Let γ be an optimal assignment in problem P1. A symmetric optimal dual solution $\mu^a = \mu^b \equiv \mu$ exists, and the prices, gains from trade, and trading rates*

$$p^b(x, \tilde{x}) = V(z(x), k(x) + k(\tilde{x})) - V(x) - \mu(x) - c_T \tag{15}$$

$$p^s(x, \tilde{x}) = V(x) + \mu(x) - V(z(x), 0) \tag{16}$$

$$V_{trade}(x) = \mu(x) \tag{17}$$

$$\lambda^b(x, \tilde{x}) + \lambda^s(x, \tilde{x}) = \frac{\gamma(x, \tilde{x}) + \gamma(\tilde{x}, x)}{\phi(x)} \tag{18}$$

for all $x, \tilde{x} \in \mathcal{X}$—with (18) *applying to states with $\phi(x) > 0$—constitute admissible trading arrangements given (ϕ, V), with the buyer in each match determined by the maximum in S. (ii) Conversely, for any admissible trading arrangements given (ϕ, V), the induced assignment $\gamma(x, \tilde{x}) = \frac{1}{2}\phi(x)[\lambda^b(x, \tilde{x}) + \lambda^s(x, \tilde{x})]$ solves* P1, *(V_{trade}, V_{trade}) is a symmetric optimal dual solution, and the trade plan attains the maximized surplus: its joint continuation-value gain net of transaction costs per market-access opportunity equals $Q(\phi, V) = \int V_{trade}(x) \, \phi(x) \, dx$.*

Theorem 1 states that the assignment problem *P1* identifies the trading allocation—who trades

with whom—and the gains from trade on its support, together with a class of decentralizing prices built from its multipliers, and that it does so without loss of generality. Part (i) is the analogue of the second welfare theorem for the trading block: the solution of the planner's problem *P1* is decentralized by prices built from the multipliers on (13) and (14). Part (ii) is the analogue of the first: any trading arrangements consistent with equilibrium maximize the surplus from reallocating capital, and the equilibrium gains from trade recover the multipliers of *P1*.

The optimality and decentralization of assignment problems are well studied, but we find it useful to review the intuition in our context.[9] The envelope theorem applied to problem *P1* implies that the social gain from having more owners with state x in the market for capital is $\mu^a(x)/2 + \mu^b(x)/2$. Because the surplus is symmetric and the same owners populate both sides of the assignment, swapping the two multipliers preserves optimality, and averaging any optimal dual pair with its swap yields the symmetric solution μ that captures the social gains. Rearranging equation (15) hows how the equilibrium prices align the social gains to the private gains. Equation (18) converts the optimal assignment into trading rates.

4.2 Characterizing (ϕ, V) given $(p^j, k^j, \lambda^j, V_{trade})$

In the second step, we use the outcomes of the first step to update value functions, V, and the invariant measure, ϕ. The characterization in the first step gives us a handy way of solving the Bellman equation. Given the value of $V_{trade}(x)$ from the multiplier μ that is obtained as part of the solution of problem *P1*, the HJB can be solved using standard methods (for example, finite differences as in Achdou et al. (2022)). The policy functions for investment (i), trades (λ^j), and entry or exit (ι^{in}, ι^{out}) govern the law of motion of the distribution for which we find a stationary point $\dot{\phi} = 0$. Together the two steps characterize the recursive competitive equilibrium as a fixed point. This characterization naturally lends itself to a computational algorithm where we iterate between the steps until convergence.

4.3 Properties of the Equilibrium

In this section, we establish three properties of the equilibrium: the trading arrangements are pairwise stable, the price of a business depends only on the quantity of capital sold, and the equilibrium allocation is efficient. We conclude by discussing extensions of the framework in which efficiency would fail.

Stability. The first property is a well-known feature of the optimal-assignment representation of competitive equilibria under transferable utility: no pair of agents can jointly gain by deviating from the proposed allocation.

[9] See Galichon (2016) for details on the Monge-Kantorovich transportation problem.

Corollary 1. *The equilibrium trading arrangements are pairwise stable: given V, no pair $(x, \tilde{x}) \in \mathcal{X}^2$ can deviate to a feasible trade that leaves both owners weakly better off than their equilibrium gains from trade, with at least one strictly better off.*

Prices. The next result builds on the price equations (15) and (16) in Theorem 1 and further sharpens the characterization of the seller's price function p^s. Call a sale from an owner with state x to an owner with state $\tilde{x}$ *executed* if it has positive transaction mass, $\phi(x)\lambda^s(x, \tilde{x}) > 0$.

Corollary 2. *There exists a function $\mathcal{P} : K \to \mathbb{R}$ such that, for every executed sale from an owner with state x to an owner with state $\tilde{x}$,*

$$p^s(x, \tilde{x}) = \mathcal{P}(k(x)).$$

Moreover, the off-path prices can be completed, without changing the allocation, trading rates, or gains from trade, so that this equality holds for every potential counterparty of every owner who sells with positive probability.

This corollary says that, on executed transactions, prices depend only on the quantity sold. The intuition for this result is straightforward. The seller's value from trade is equal to the price he extracts from the buyer plus the value of starting anew with zero capital and the current level of productivity. The second component is independent of the trading partner. Thus, if the same seller executes sales to two different buyer types—each with positive transaction mass—she must receive the same price in both transactions. A similar argument from the perspective of the buyer shows that executed prices do not depend on the seller's productivity. Off-path prices, by contrast, are not pinned down by equilibrium: under the buyer-based price selection of Theorem 1, a would-be seller may be offered strictly less than the executed price. The second claim in the corollary is a permissible normalization: off-path prices can be reset to the same schedule without changing the allocation, trading rates, or gains from trade, so the imputed market price of unsold businesses with capital k can be set according to $\mathcal{P}(\cdot)$.

While our general formulation of pairwise terms of trade allows for arbitrary gains from matching with any owner that has type $\tilde{x}$, our assumption that productivity z is non-transferable delivers an equivalence between our trading protocol for capital, and a competitive market with stochastic access at rate η and unit demand over differentiated products, which are indexed by the indivisible size of the capital sold.[10]

[10]Our model can easily accommodate extensions in which the gains from purchasing capital depend on the productivity of the seller. Among other things, this feature could capture instances when the sale consideration includes a consulting contract with the seller in order to facilitate the transition to new ownership. See Bhandari and McGrattan (2021) for details.

Efficiency. We now turn to the efficiency properties of our competitive equilibrium. Given ϕ_0, consider a planner that solves the following maximization problem

$$P(\phi_0) = \max_{\{n_t, b_t, i_t, \iota_t^{\text{in}}, \iota_t^{\text{out}}, \lambda_t^j, k_t^j\}} \int_0^\infty e^{-\rho t} \int \left[y(x, b_t, n_t) - (\rho + \delta_b) b_t(x) - c(i_t(x)) - \eta c_T \int \lambda_t^b(x, \tilde{x}) d\tilde{x} \right] \phi_t(x) dx \text{dt}$$

such that the time-dependent analogues of the law of motion for the distribution of owners (7), labor-market feasibility (9), the indivisibility restriction (1), the consistency of meeting probabilities (11), and the individual trading-probability constraint in (3) are all satisfied, with control domains $n_t, b_t, i_t \geq 0$, $\lambda_t^j \geq 0$, and $\iota_t^{\text{in}}, \iota_t^{\text{out}} \in \{0, 1\}$.[11] The state path $\{\phi_t\}_{t \geq 0}$ consists of nonnegative finite measures satisfying the law of motion and the initial condition ϕ_0. We label this problem as *P2*. Given linear preferences, maximizing discounted welfare is the same as maximizing discounted output net of investment and transaction costs.

In the next theorem, we show that a stationary recursive equilibrium is efficient.

Theorem 2. *A stationary recursive equilibrium with the stationary measure ϕ achieves $P(\phi)$ in problem P2.*

The forces towards efficiency were foreshadowed in Theorem 1: given (ϕ, V), equilibrium trading maximizes the surplus from reallocating capital, and the gains from trade are the multipliers of *P1*. Beyond the static assignment, problem *P2* has additional margins—entry, exit, labor, and investment—and its comparison set includes nonstationary plans. The proof is a verification argument: the equilibrium value functions generate an exact upper bound on the planner's payoff along every feasible path, and the equilibrium path attains it.

We conclude our discussion of the efficiency properties of equilibrium by emphasizing that our framework is flexible enough to incorporate extensions, likely to be relevant for private firms, that may generate inefficiencies. For example, consider the case where business capital is rival across firms: the customer base or supplier relationships that one owner builds may come partly at the expense of those of other firms, so that investment in this type of capital entails business stealing. A natural way to capture this feature in our setting would be to let the depreciation rate of business capital, δ_k, depend on aggregate investment. Intuitively, when many firms invest heavily, the returns to any one firm's past investments erode more quickly. As a second example, consider a setting in which firms produce differentiated goods and compete with market power. Our framework can be extended to allow for size-dependent markups that give larger firms a competitive advantage. In this case, acquisitions could alter efficiency both directly, by changing the distribution of firm sizes, and indirectly, by shifting the extent of product-market power. We view both extensions as important and believe these mechanisms could be disciplined with appropriate data and quantified in future work.

[11] To avoid additional notation, the planner rents fixed assets from outside the economy at the user cost $\rho + \delta_b$; up to the value of the initial stock, this is equivalent to having the planner accumulate fixed assets subject to depreciation.

Next, we describe the firm-level data used to parameterize the model.

5 Data

In this section, we describe the main datasets and the moments based on IRS administrative tax records that we use to calibrate the parameters of our model.

5.1 IRS Samples

There are two main firm-level databases that we use at the IRS. The first database includes transcribed items from business tax filings for the universe of businesses that file Form 1120 as Subchapter C corporations, Form 1120-S as Subchapter S corporations, and Form 1065 as partnerships. We complement this database with the universe of electronically-filed returns that contains information on business sales recorded on IRS Form 8594 (*Asset Acquisition Statement Under Section 1060*) when there is a transfer of business assets that make up a trade or business for either the seller or the buyer.[12]

When a business is sold, the IRS must be informed about the allocation of the purchase price across different asset categories. Form 8594 groups the assets transferred into seven classes: cash (Class I); actively traded securities (Class II); accounts receivable and debt instruments (Class III); inventory (Class IV); fixed assets such as equipment, buildings, and land (Class V); intangible assets defined in Section 197 of the Internal Revenue Code other than goodwill and going concern (Class VI); and goodwill and going-concern value (Class VII). The Section 197 intangibles include assets such as customer bases, trade names, franchises, licenses, and covenants not to compete. The allocation of price across assets is relevant for the seller, who must report capital gains, and the buyer, whose cost recovery is governed by the asset class and in particular, Section 197 intangibles are amortized over 15 years. In a typical sale, the parties' incentives conflict: the seller prefers to allocate more of the price to intangibles such as self-created goodwill, whose gain is a long-term capital gain, while the buyer prefers to allocate towards assets with faster cost recovery, such as equipment and inventory. This tension, together with the requirement that both parties report the same allocation on Form 8594, contributes to the reliability of the reported allocations.[13]

In our quantitative analysis, we focus on corporate businesses that elect Subchapter S status for tax purposes and their counterparties across legal form status in business sales. Under Subchapter S, profits and losses flow through the corporation untaxed and are taxed directly as income to the owner on their individual tax forms. S corporations are now the most prevalent type of cor-

[12] See `https://www.irs.gov/forms-pubs` for details on these tax forms.

[13] In many cases, the buyer's amortization basis in particular assets is determined only by the amount paid at the time of the sale. This is typically the case for Section 197 intangibles and goodwill, whose values are rarely observed outside of a sale.

Table 1: IRS SAMPLES

BUSINESS SAMPLES	COUNTS
S corporation population	3,167,266
Seller-buyer pairs:	51,286
S Corporation − S Corporation	28,078
− Partnership	14,040
− C Corporation	9,168

Notes: The 'S corporation population' is drawn from the universe of S corporation filings over the period 1996–2022 and excludes any firms with wage bill under $10,000 or insufficient data for constructing a three-year growth rate in the wage bill. This set of firms is our *full sample*. The 'Seller-buyer pairs' are S corporation sellers found on e-filed Form 8594 that have available data on their wage bill in the year prior to the sale and buyer counterparties that have available data on their wage bill in the year after the sale. These firm pairs are our *trading sample*.

poration in the United States, accounting for more than three-quarters of all corporate tax filings. Unlike other corporate forms, S corporations can have at most 100 owners (typically only 1 or 2), and the owners must be U.S. citizens or permanent residents. The fact that S corporations are owned by individuals is relevant for our analysis as we are interested in capital transfers between owners that actively manage their businesses. In contrast, most owners of C corporations are passive investors that rely on oversight from boards of directors. Subchapter C corporations and partnerships do not have the same ownership restrictions: C-corporate shareholders or partners can be businesses. However, we record information for these business types if they are counterparties in a sale of an S corporation.[14]

The focus on S corporations does not change the picture of physical capital intensity documented in Section 2. Using aggregate balance sheets and income statements for S corporations published in U.S. Internal Revenue Service (2022), the book-value capital-output ratio for tax year 2022 is 0.36. Revaluing the historical-cost stocks using BEA price indexes and depreciation, which makes them comparable to the current-cost measures used in economy-wide estimates, raises the ratio to 0.49. As we discussed in Section 2, these capital-output ratios are well below the economy-wide benchmark of 3.3 cited in the macroeconomic literature.

In Table 1, we record counts for two IRS data samples. The first sample is the universe of S corporations over the period 1996–2022 with $10,000 or more in wage bill. We also require that the firms in this sample have sufficient data to construct growth rates of the wage bill over a 3-year period. These rates use information for the current tax year and three years prior. We use wage bill

[14]Ideally, we would include sole proprietorships in our analysis of sellers. However, the electronically-filed database that contains Form 8594 does not include sole proprietorship filings.

Table 2: INTANGIBLE INTENSITIES
IRS S CORPORATION TRADING SAMPLE

INTANGIBLE INTENSITIES	PERCENTILES 25th	50th	75th
Sales to S corporations	29.2	66.8	87.0
Partnerships	33.3	71.0	90.9
C corporations	39.3	73.4	92.7
All sales	32.1	69.3	89.3

Notes: See Table 1 for details on the trading sample. The intangible intensity is the ratio of intangible assets (Section 197 assets plus goodwill) to the sum of intangible and tangible assets (inventories, fixed assets, real estate) listed on Form 8594. To ensure that no confidential information is disclosed, reported percentiles are computed as an average of observations around the value listed in the table.

to measure firm size because other measures such as income and profits can be easily manipulated using tax evasion strategies. Hence the focus on employer firms using the cutoff of $10,000 in wage bill. With these restrictions, we have panel data for a total of 3.2 million S corporations. We refer to this set of firms as the *full sample*.

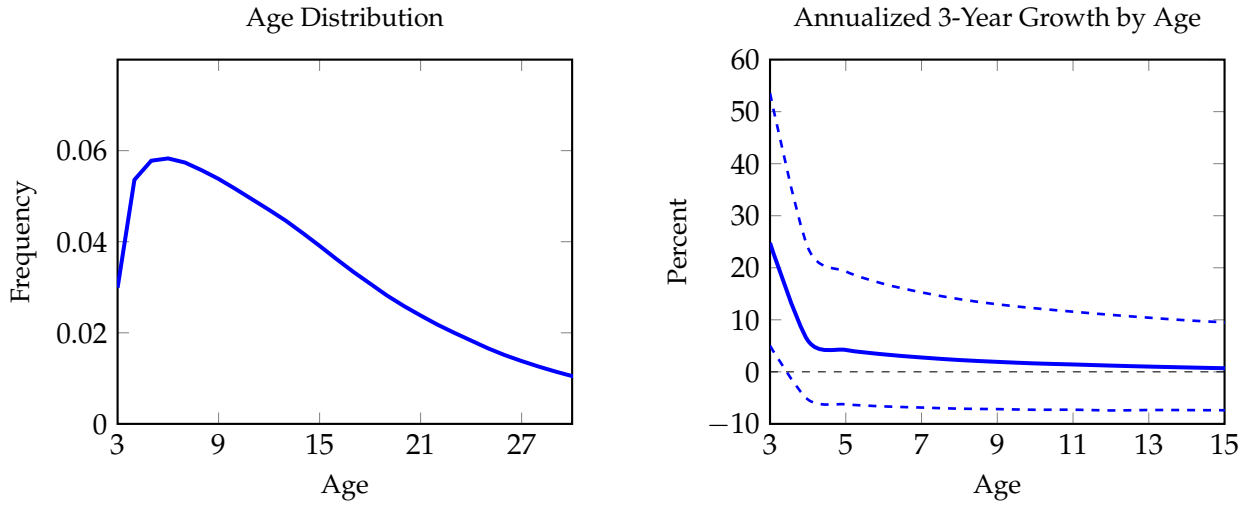
The second sample in Table 1 is our *trading sample*. The trading sample includes pairs of S corporations from the full sample and buyers found on the e-filed Forms 8594 over the period 2005–2022. We restrict attention to pairs with available data on their business tax filings to construct measures of relative size. More specifically, we construct a sample of seller-buyer pairs for which we have information on the seller's wage bill in the year prior to the sale and the buyer's wage bill in the year after.[15] We restrict attention to sales between employer firms and therefore also include a restriction that the buyer has a wage bill over $10,000. The trading sample constitutes only a subset of all S corporation asset sales, but contains essential moments for our calibration.[16]

Auxiliary data from brokered business sales and IRS published sources are used to inform some model moments that are not easily identified using the samples described above. In the case of brokered sales, we have a sample of 6,858 transactions by legal form from Pratt's Stats (currently DealStats) for the period 1994–2017. These data include the purchase price allocations that appear on IRS Form 8594, some pre-sale financial statistics, the business age, and details about the listing.

[15]Not all parties that are involved in a business sale file a Form 8594. Partial information on business sales for which we have no Form 8594 should appear for the selling owners as a capital gain on Schedule D, but detailed information on the assets is not available.

[16]While the model does not feature partial sales, the sample can include them: Form 8594 covers transfers of asset groups that constitute a trade or business for at least one party, including sales of a division or line of business after which the seller continues to operate. Restricting the trading sample to transactions in which the seller does not continue to operate after the sale leaves the main calibration moments largely unchanged.

Figure 1: AGE AND GROWTH DISTRIBUTIONS
IRS S CORPORATION FULL SAMPLE



Notes: Both panels use the IRS S corporation full sample described in Table 1: the universe of S corporation filings over 1996–2022 with a wage bill of at least $10,000 and sufficient history to construct a three-year wage-bill growth rate. The left panel plots the frequency distribution of business age, measured in years from the establishment date. The right panel plots percentiles of annualized three-year wage-bill growth by age: for each age, growth is the percentage change $[(wb_t/wb_{t-3})^{1/3} - 1] \times 100$, computed from the current tax year and three years prior; the solid line is the median, the dashed lines are the 25th and 75th percentiles, and the horizontal dashed line marks zero growth.

Important information in the Pratt's Stats dataset, not available in IRS filings, includes the date of listing and the date of sale. These dates are used to inform the model's time to trade. In the case of IRS published sources, we use business entity data provided by the Statistics of Income (SOI) program to measure cost shares in value added for S corporations.

5.2 Intangible Intensity

We use transaction prices between seller-buyer pairs in our trading sample from Table 1, and construct the ratio of the purchase price allocated to Section 197 intangibles and goodwill (Classes VI and VII on Form 8594) to the purchase price allocated to Classes IV through VII, which include inventory, fixed assets, real estate, intangibles, and goodwill. Classes I through III, which include cash and marketable securities, are excluded. We call this ratio the *intangible intensity*.

In Table 2, we report moments of the intangible intensity distribution.[17] The median intangible intensity is 69 percent if we consider all sales, but is hardly different across groups of buyers categorized by legal form.[18] As discussed in Section 2, these shares are lower bounds for business

[17] Here and below, we compute percentiles as an average of observations in a window around corresponding percentile values listed in the table. This computation ensures that no confidential taxpayer information is disclosed.

[18] Bhandari and McGrattan (2021) find similar results using the Pratt's Stats database. See their appendix for more details and analysis of different subpopulations in the data.

capital with the properties we model: many assets reported as "fixed assets" (Class V) on Form 8594 are customized to the business and are similarly difficult to rent, divide, or redeploy. Including them along with the Section 197 intangibles would raise the intangible intensities in Table 2 even further.

5.3 Empirical Moments

We next describe the data moments that will guide our model calibration. These moments include information on business age and growth, as well as estimates of business valuation and relative size of buyers and sellers.

Using dates of business establishment, we compute a business age for each S corporation in our IRS full sample. In the left panel of Figure 1, we plot the age distribution for these corporations, which starts at age three given our sample construction and peaks around age six. In the right panel, we plot the distribution of annualized 3-year growth rates for the IRS full sample, which are constructed from wage bills. Here, we plot the 25th, 50th, and 75th percentiles at each age. As the figure shows, the median growth is 25 percent initially and falls to 6 percent the following year. This declining age-growth profile is also documented in other studies of firm-level data (see, for example, Haltiwanger et al. (2013)). At young ages, the interquartile range of wage bill growth is roughly twice the median, and it narrows as firms mature.

In the top panel of Table 3, we report summary statistics for our full sample. The first rows are population moments for the data plotted in Figure 1. The interquartile range of business ages for this sample is 8 to 21 years. The interquartile range of wage growth is -7 to 12 percent. We also report these percentiles for the log of the wage bill in the case of young firms (age-3 firms in the IRS full sample). Comparing the interquartile ranges, we do not find large differences between the statistics for the age-3 firms and those for all ages of the population. We see in Figure 1 that growth is rapid only in the few years following the establishment of the business, and thus we should not be surprised to find small differences between young firms and the broader population.

The lower panels of Table 3 report key statistics from our IRS trading sample. We first report valuation multiples, which are computed as the ratio of the total price paid for a group of business assets to the wage bill of the seller in the year before the sale. We report sales to S corporations, C corporations, and partnerships separately since C corporations and partnerships owned by other business entities tend to be larger in size. Roughly half of the counterparties in sales involving S corporations are S corporations themselves. For the median sales across the three categories of exchange, we find a range of valuation multiples of 2.4 to 4 times the seller's wage bill.

In the bottom panel of Table 3, we report statistics for a measure of relative size: the ratio of the wage bill of the buyer a year after the sale to the wage bill of the seller a year before the sale. As compared to valuation multiples, we find much more heterogeneity in relative sizes. For our

Table 3: SUMMARY STATISTICS
IRS S CORPORATION FULL AND TRADING SAMPLES

STATISTIC	PERCENTILES 25th	50th	75th
FULL SAMPLE			
Business Age (years)	8.0	13.0	21.0
Wage Growth (percent)	−6.8	1.4	11.6
Log Wage Bill: Entrants	10.9	11.6	12.4
Population	10.9	11.8	12.7
TRADING SAMPLE: VALUATION MULTIPLES			
Sales to S corporations	1.0	2.4	5.2
Partnerships	1.4	3.5	8.6
C corporations	1.5	4.0	9.9
All sales	1.2	2.9	6.7
TRADING SAMPLE: RELATIVE WAGE BILL SIZES			
Sales to S corporations	0.7	1.4	5.6
Partnerships	1.0	2.8	17.4
C corporations	2.2	14.9	130.7
All sales	0.9	2.1	13.5

Notes: See Table 1 for details on the full and trading samples. In the full-sample panel, "Entrants" refers to businesses at age three, the youngest age at which the sample's three-year growth-rate requirement can be met, so the row describes age-three survivors rather than firms at their first filing; wage growth is the annualized three-year growth rate of the wage bill, computed as the percentage change $[(wb_t/wb_{t-3})^{1/3} - 1] \times 100$; log wage bills are natural logarithms of wage bills measured in 2012 dollars. In the trading-sample panels, the "valuation multiple" is the ratio of firm sale price to the seller's wage bill in the year prior to the sale, and the "relative wage bill size" is the ratio of the buyer's wage bill in the year after the sale to the seller's wage bill in the year prior to the sale. To ensure that no confidential information is disclosed, reported percentiles are computed as an average of observations around the value listed in the table.

sample of S corporations, the median size ratios across the different legal form categories range from 1.4 times the seller's wage bill, where the buyers are S corporations, to 15 times, where the buyers are larger C corporations.[19]

Because there is a wide range of values for the relative wage bill size by legal form, we also report information on this key statistic by the size of the S corporation seller. More specifically, we

[19]When parameterizing the model, we use a value-weighted estimate for sales to S corporations and partnerships rather than the equal-weighted "all sales" measure to avoid having the estimates dominated by the sale of relatively small firms to large publicly-traded companies.

Table 4: WAGE BILLS OF SALE COUNTERPARTIES
IRS S CORPORATION TRADING SAMPLE

SELLER WAGE BILL BIN RANGE		BUYER WAGE BILL PERCENTILES		
LOWER	UPPER	25th	50th	75th
60	99	82	211	1,167
99	163	121	268	1,794
163	268	178	377	2,758
268	442	285	667	5,076
442	729	460	1,290	9,436
729	1,203	790	2,304	13,798
1,203	1,983	1,303	3,402	17,022
1,983	3,269	1,830	4,593	22,748

Notes: See Table 1 for details on the trading sample. All wage bills are reported in thousands of 2012 dollars. Sellers' wage bills one year prior to the sale of the business are assigned to ten bins of width 0.5 in logs, which are reported in thousands of 2012 dollars in the table; the table displays the eight interior bins. The columns report percentiles of the wage bills in the year after the sale for buyers who were counterparties on Form 8594 to sellers in the bin. To ensure that no confidential information is disclosed, reported percentiles are computed as an average of observations around the value listed in the table.

take the log of the wage bills for all sellers in the IRS trading sample and assign them to 10 bins. For each bin, we compute percentiles of the log wage bills of the buyers who are counterparties to the sales reported on Form 8594. In Table 4, we report the results for seller log wage bills between 11 and 15, corresponding to a range between about 60 thousand and 3.3 million dollars. Each row covers a bin of sellers, and the columns show the 25th, 50th, and 75th percentiles of the buyers' wage bills. Note that the 25th percentile of the buyer wage bill is similar in magnitude to the seller's own bin. This means that most buyers are larger in size than the sellers. The 75th percentile buyer is about an order of magnitude larger than the typical seller. Also notable is the clear pattern that emerges: such size gaps between buyers and sellers are nearly constant across the seller's size distribution.

6 Calibration

In this section, we report on our model parameters and model fit. We first specify functional forms for the primitives of the model. Some parameters are then pre-set using U.S. aggregate statistics and other moments that map into the model primitives directly. The remaining parameters are

Table 5: MODEL PARAMETERS

PARAMETER	EXPRESSION	VALUE
PRE-SET PARAMETERS		
Discount rate	ρ	0.050
Entry/death rate	ψ_e	0.025
Occupation-switch rate	ψ_o	1.00
Rentable capital share	β	0.35
Labor share	θ	0.35
Depreciation of rentable capital	δ_b	0.041
Depreciation of transferable capital	δ_k	0.058
Trading arrival rate	η	2.17
ESTIMATED PARAMETERS		
Investment cost scale	A	4.312
Investment cost curvature	χ	4.706
Transferable capital share	α	0.115
Transferable capital volatility	σ_k	0.560
Entrant productivity rate	ν	3.053
Productivity volatility	σ_z	0.048
Productivity mean	$\bar{z}$	0.841
Productivity mean reversion	κ_z	0.140
Exogenous destruction rate	δ_{exo}	0.020
Per-trade resource cost	c_T	0.50

Notes: The production function is given by $y = \exp(z)\, k^{\alpha} b^{\beta} n^{\theta}$. The non-transferable productivity process is given by $\mathrm{d}z = \kappa_z(\bar{z} - z)\mathrm{d}t + \sigma_z \mathrm{d}\mathcal{W}_z$. The cost function for capital investment is $c(i) = [\exp(A\chi) i^{1+\chi}]/(1+\chi)$. The capital accumulation of transferable capital is governed by $\mathrm{d}k = (i - \delta_k k)\mathrm{d}t + \sigma_k k \mathrm{d}\mathcal{W}_k$. The initial productivity is drawn from the distribution $1 - \exp(-\nu z)$, $z \geq 0$.

estimated by simulated method of moments utilizing the moments discussed in Section 5.3 that were constructed from the IRS data.

6.1 Functional Forms

The quantitative analysis requires parametric choices for the production technology, the productivity process, the investment cost, and the entrant productivity distribution. Output is produced with the Cobb-Douglas technology

$$y(x, b, n) = \exp(z(x))\, k(x)^{\alpha} b^{\beta} n^{\theta},$$

where $\alpha + \beta + \theta < 1$. Productivity follows an Ornstein-Uhlenbeck process, with drift $\mu_z(z) = \kappa_z(\bar{z} - z)$ and volatility $\sigma_z(z) = \sigma_z$, where κ_z is the rate of mean-reversion, $\bar{z}$ is the long-run mean, and σ_z is the innovation volatility. The cost of investment is

$$c(i) = \frac{e^{A\chi}}{1+\chi} i^{1+\chi},$$

where A is the cost shifter and $\chi > 0$ is its curvature. Finally, the productivity of entrants is drawn from an exponential distribution, $G_z(z) = 1 - \exp(-\nu z)$ for $z \geq 0$, with parameter ν.

6.2 Pre-Set Parameters

In Table 5, we report the model parameters. In the top panel of the table, we report values for the pre-set parameters. The model period is one year, matching the annual tax-reporting cycle. We set the discount rate ρ to 5 percent. The rate of new births and deaths is ψ_e, which is set equal to 2.5 percent to represent an average working life of 40 years. The parameter ψ_o governing the option to switch occupations is set to 1, so individuals receive an occupational-switch opportunity once per year on average. The pre-set production shares for external factors (rentable capital and labor) are informed by revenue shares reported separately for S corporations in the SOI's *Corporation Income Tax Returns Complete Report* (U.S. Internal Revenue Service, 2022). See, for example, Table 6.1 of the report for tax year 2022. The values we use are $\beta = 35$ percent for rentable capital and $\theta = 35$ percent for labor. Values for depreciation rates are informed by capital obsolescence studies conducted by the BEA and service lives of intangibles reported by U.S. General Accounting Office (1991). These imply values of $\delta_b = 4.1$ percent and $\delta_k = 5.8$ percent. (See Bhandari and McGrattan (2021) for more details.)

The last pre-set parameter is key for the potential meeting frequency. The arrival rate η is set equal to 2.17, which is consistent with a trading opportunity occurring roughly twice per year. As noted earlier, this estimate is based on the median time between the date a business is listed and the date of its sale reported in the Pratt's Stats brokerage data.

6.3 Estimated Parameters

The parameters reported in the lower panel of Table 5 are:

$$\Theta = [A, \chi, \nu, \sigma_z, \kappa_z, \bar{z}, \alpha, \sigma_k, \delta_{\text{exo}}, c_T]$$

and they are estimated by simulated method of moments. Most of these parameters are traditionally estimated with observations using firm or plant-level data on measures of value added, investment expenditures and capital retirements.[20] Crucially, the relevant capital here is unobserved until a business is sold, making traditional methods not applicable in our context. Instead,

[20]Olley and Pakes (1996) and Cooper and Haltiwanger (2006) are notable examples that use these data to estimate production function and adjustment cost parameters, respectively.

we rely on an estimator $\hat{\Theta}$ that minimizes weighted squared percentage deviations between model and IRS moments related to firm dynamics and the novel relative size and valuation multiples described in Section 5.[21]

The estimation results are reported in the last column of the lower panel of Table 5. To build intuition for identification, it is useful to distinguish three sets of moments: the trading moments (the valuation ratio and the relative size of buyers and sellers); the firm-dynamics moments (the dispersion in entrant size, the level and dispersion of growth rates by age, and the mass of business owners); and other moments (the mean and median business age and the average intermediation fee).

The trading moments sort the production function and investment cost related parameters in Θ into several clusters, according to whether a parameter moves the two trading moments in opposite directions, moves them in the same direction, or moves the valuation ratio alone. The firm-dynamics moments then separate the parameters within each cluster. The remaining parameters are identified by auxiliary moments such as the mean and median business age and the average intermediation fees.

The investment cost parameters, A and χ, move the trading moments in opposite directions. When building capital internally is expensive, acquiring it through trade is the cheaper margin: owners bid up the prices of existing businesses, which raises valuation ratios, and productive buyers grow less through internal investments, lowering the relative size of buyers to sellers. The productivity parameters ν, σ_z, κ_z, and $\bar{z}$ move the trading moments in the same direction because they govern, in various ways, the dispersion of productivity across firms and the extent of mismatch between productivity and capital. Consider the entrant rate parameter ν. A lower ν thickens the right tail of entrant productivity, creating more high-productivity owners with little capital, the natural buyers. With greater heterogeneity of this kind, the typical buyer is more productive than the typical seller: buyers pay more for a given business, raising valuation ratios, and are larger relative to the sellers they buy from, raising relative size. Finally, the capital parameters, α and σ_k, are characteristics of the capital itself rather than of the owners who trade. The output elasticity α raises every owner's marginal product of capital, and the volatility σ_k alters the risk of holding capital. Both change what capital is worth, and hence the valuation ratio, but not who trades with whom, leaving the relative size of buyers and sellers largely unaffected.

The firm-dynamics moments then distinguish parameters within each cluster. In the cost cluster, the scale parameter A sets the pace of internal capital accumulation and therefore the level of growth rates by age, whereas the curvature parameter χ governs how strongly investment responds to differences in marginal products and therefore the dispersion of growth rates. In the

[21] To improve numerical stability, the computational implementation of the model uses small logit smoothing terms in occupational switching and trade choice. See replication codes for more details. Optimization is performed using a multi-start, derivative-free simplex algorithm on a bounded parameter space. We weight the trading moments more heavily than the firm-dynamics moments to ensure that the model matches the trading data well.

productivity cluster, moments of young firms separate the entry margin from the long run: the dispersion in entrant wage bills and growth at early ages are informative about the entrant distribution parameter ν and the mean-reversion rate κ_z, while the size and growth of mature firms are governed by the long-run mean $\bar{z}$ and the volatility σ_z. The mass of business owners, which moves nearly one-for-one with selection at entry, provides additional leverage on ν. In the capital cluster, α controls how much the dispersion in productivity among young firms turns into dispersion in growth rates, while σ_k is identified from the dispersion in growth rates among older firms, for whom productivity dispersion is relatively more compressed.

The exogenous capital destruction rate δ_{exo} helps the model match, among other things, the mean and median business age. Finally, the per-trade resource cost c_T is disciplined by the fees that intermediaries charge in private business sales: we target an average fee of 10 percent of the transaction price, in line with the commissions of 8 to 15 percent that the International Business Brokers Association reports for typical brokered sales (International Business Brokers Association, 2021).

6.4 Model Fit and Validation

In Table 6, we compare the moments used in the estimation of $\hat{\Theta}$ to the counterparts in our model. Overall, the estimated model reproduces the main moments used in estimation. The trading statistics are matched closely. Growth rates and business ages are matched with some deviations, which we note below. In the table, we report levels and interquartile ranges for ages 3 and 10, but the fit extends beyond the targeted ages: in the left panel of Figure 2, we overlay the model's growth-rate percentiles on their IRS counterparts and find that the model tracks the level and dispersion of growth across the entire age distribution. The data counterpart for the mass of owners is constructed from two facts: a self-employment rate of roughly 10 percent and a private-sector share of roughly 50 percent for wage workers. Since our model describes the private sector, in which each individual is either a business owner or a private-sector worker, the implied fraction of owners is 0.18. The main challenge is matching the observed dispersion in the log wage bill of young firms. The main parameter governing this moment is the rate parameter ν, which is also important for firm dynamics and the relative size of buyers and sellers. Richer forms of ex-ante heterogeneity could potentially address this shortcoming.

The intermediation fees in the model also line up with industry practice beyond the targeted average. Because the cost is fixed per transaction, the implied fee share is strictly decreasing in the transaction price: in our baseline equilibrium, the fee is 3 percent of the price for the transaction at the 75th percentile of prices and 13 percent at the 25th percentile. This variation is consistent with how fees are set in practice: practitioner guides describe broker success fees as a percentage of the purchase price that varies with the size and structure of the deal (Schenck, 2023), and smaller

Table 6: FIT OF THE MODEL

MOMENT	MODEL	DATA
TRADING RATIOS		
Valuation ratio, Median	3.21	3.13
Relative size, Median	2.26	2.33
Intermediation fee / sale price, Mean	0.10	0.10
ANNUALIZED GROWTH RATES		
Age 3, Median	0.26	0.25
Age 3, Interquartile range	0.36	0.49
Age 10, Median	0.01	0.02
Age 10, Interquartile range	0.20	0.20
BUSINESS AGE		
Median Age	8.31	10.00
Mean Age	11.28	12.00
MASS OF OWNERS		
Fraction of business owners	0.20	0.18
LOG WAGE BILL		
Entrants, Interquartile range	0.89	1.50

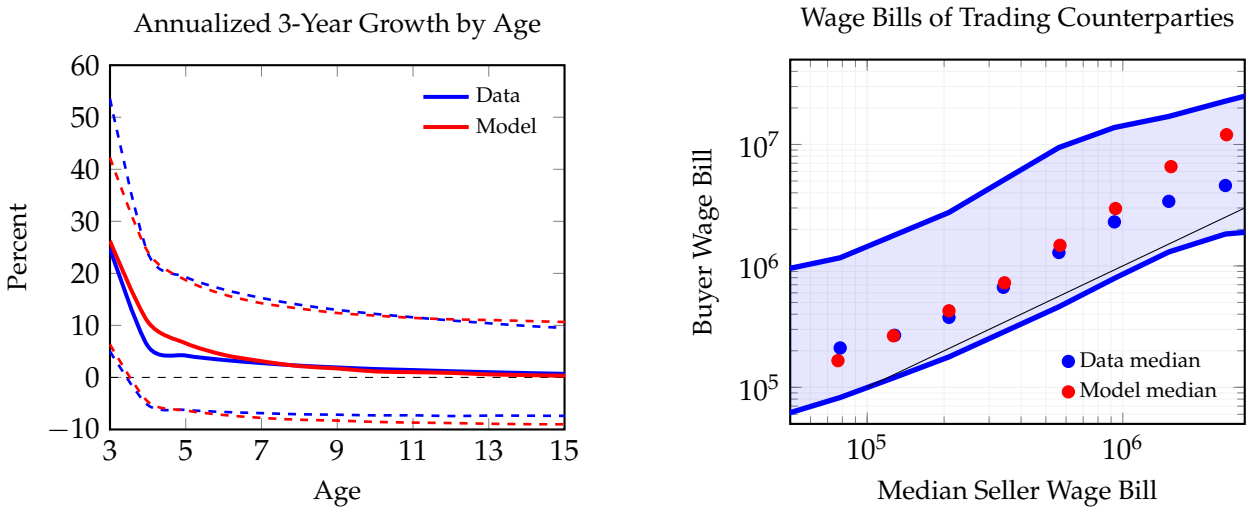
Notes: Growth rates are annualized over three years. The U.S. valuation multiple and relative size estimates are based on value-weighted statistics for S corporation sales to counterparties that are S corporations or partnerships.

percentages apply to larger transactions (International Business Brokers Association, 2021).[22] In addition to matching intermediation fees, the fixed cost of trading screens out low-surplus trades and helps match the observed frequency and selection of transactions by business age and size.

Beyond the moments we target in Table 6, the model predicts that indivisible trades generate systematic patterns in relative sizes of counterparties that are consistent with the data. In the right panel of Figure 2, we show predictions of buyer wage bills across the distribution of seller-size bins. The increasing pattern is a combination of the mechanical effect of purchasing larger firms, and the specific selection of buyers into each market segment. The data, in blue, are a visual summary of the IRS moments coming from our transaction sample and reported in Table 4. The figure shows that the model (in red) matches the data medians well, perhaps somewhat overshooting at the top.

[22]The variation in fees relative to transaction value in the model is also consistent with the sliding scales used in mid-market M&A transactions, in which the commission percentage declines with transaction value—for example, the Lehman formula and its double-percentage variant (Wikipedia, 2026).

Figure 2: GROWTH BY AGE AND WAGE BILLS OF TRADING COUNTERPARTIES
MODEL VS. DATA

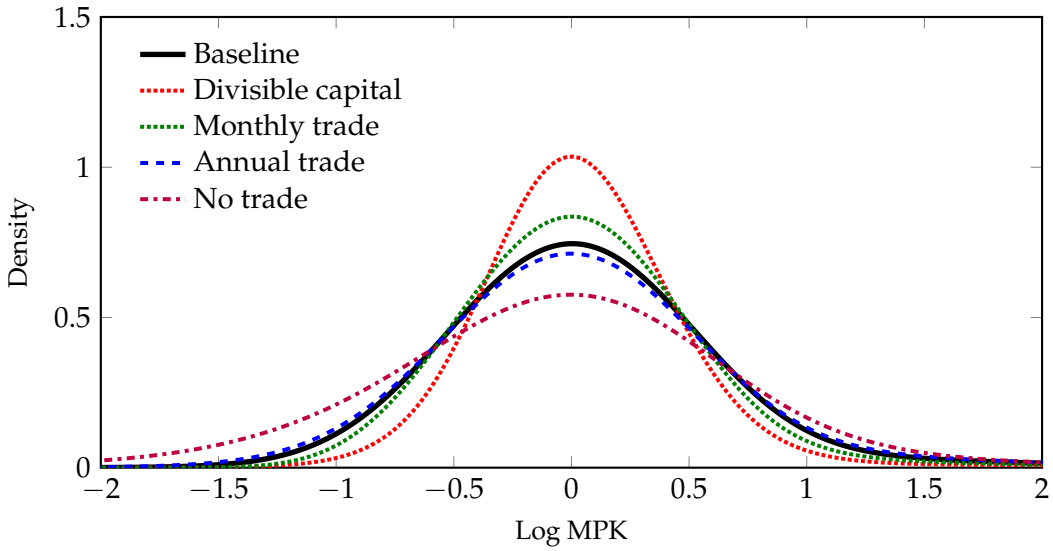


Notes: In the left panel, solid lines are medians and dashed lines are the 25th and 75th percentiles of annualized 3-year wage-bill growth rates, by business age. The data, in blue, reproduce the right panel of Figure 1 and are constructed from the IRS S corporation full sample. The model percentiles, in red, are computed from the baseline economy using the same age and growth-window definitions as the IRS sample. Only the medians and interquartile ranges at ages 3 and 10 are targeted in the estimation. In the right panel, the shaded region is the interquartile range of buyer wage bills in the year after a sale with an S-corporate counterparty in the IRS S corporation trading sample (see Table 4); blue and red markers show the median buyer wage bill within each seller-size bin for the data and model, respectively. To ensure that no confidential information is disclosed, reported percentiles for the IRS data are computed as an average of observations around the corresponding values.

7 Model Predictions

In this section, we report on key model predictions. Several of these predictions have no counterpart in U.S. data on private businesses, but they are relevant for understanding capital allocation and dynamics in private businesses, and they serve as inputs to our tax policy analysis. The first is the dispersion in the marginal product of capital, which is a central statistic in the literature on capital misallocation. We compute this dispersion in our baseline and show how it changes as we vary the time to trade and allow for perfect divisibility of capital. The second is the distribution of investment rates, jointly determined by internal capital accumulation and by the reallocation of businesses across owners. In the context of physical capital, the empirical counterpart of this distribution has been central to the literature studying the technology of firm investment. The third set of statistics relates to private-business wealth. Partial information on this wealth is typically available only when a business is transferred, yet it is central to the literature on wealth inequality. We report the model's predictions for income yields, the share of value that is transferable, and the ratio of total private-business value to output.

Figure 3: PREDICTED DISTRIBUTIONS OF LOG MPK
VARYING DIVISIBILITY AND TIME TO TRADE



Notes: MPK is the marginal product of capital given by $\alpha y(x)/k(x)$. The distributions of log MPK are constructed using the universe of model corporations with sufficient data to construct a three-year growth rate in wage bill. In the baseline model, we assume indivisible trade with $\eta = 2.17$. Monthly trade assumes $\eta = 12$; Annual trade assumes $\eta = 1$; and no trade assumes $\eta = 0$.

7.1 Dispersion in Marginal Product of Capital

In a neoclassical benchmark economy with continuously accessible rental markets and perfectly divisible capital, the marginal products of capital, $\alpha y(x)/k(x)$, are equalized across firms. Because we depart from these features, our model generates dispersion in marginal products. We plot the distribution of model-generated $\alpha y(x)/k(x)$ in Figure 3 for the baseline parameterization and for alternative cases to isolate the role of our departures from the neoclassical benchmark.

Starting with the baseline parameterization (the solid line in Figure 3), we find significant heterogeneity across firms. The distribution has a standard deviation of 57 log points (that is, 100 times the standard deviation of log MPK). Two features of the model generate this dispersion: time to trade and indivisibility in capital exchange. The time to trade limits the speed at which firms can access the capital market, while the indivisibility constrains the set of feasible capital allocations within each trading pair.

In Figure 3, we plot results for alternative economies to highlight the role of trading frequency and indivisibility. Consider the trading frequency first. In the baseline calibration, we set η equal to 2.17 to replicate a median time to sale of 168 days based on the Pratt's Stats listings. To see how the dispersion changes as we vary the trading frequency, we consider three alternatives: once per month ($\eta = 12$), once per year ($\eta = 1$), and no trade at all ($\eta = 0$). The results are shown

alongside the baseline in Figure 3 (and labeled "Monthly trade," "Annual trade," and "No trade," respectively). Without trade, the standard deviation is higher, roughly 76 log points. Interestingly, with very frequent trade, in this case monthly, the standard deviation is still high, roughly 49 log points. Comparing this to an annual frequency, with a standard deviation of 61 log points, we conclude that the standard deviation is relatively flat in the empirically relevant region of values for η.

Next, we investigate the role of indivisibility by considering a version of the model in which firms can trade any amount of capital with each other given an opportunity to trade. This implies that the marginal value of capital is equalized within each trading pair. The rest of the trading protocol is left unchanged, so that trading remains bilateral and occurs at stochastic intervals.[23] If we keep the frequency of trade at its baseline value, we find some decrease in the standard deviation, to roughly 43 log points, but the indivisibility does not dramatically alter the capital allocation. When firms rarely meet potential trading partners, improving the efficiency of an individual trade has only a modest effect on the aggregate allocation. This point is starkest at $\eta = 0$, in which case divisibility is irrelevant. If trading opportunities are frequent, then divisibility becomes salient: shrinking firms are able to downsize their capital holdings smoothly, and expanding firms are less concerned with hedging against the risk of future negative productivity shocks. From this we conclude that the dispersion in marginal products is determined by the interaction between the frequency of trading opportunities and the indivisibility of capital, rather than by either feature in isolation.

Our findings on the dispersion in the marginal product of capital cannot be directly compared to studies in the literature, as no such empirical measure exists, but should serve as a model-based reference for the case of non-rentable, indivisible capital in private businesses. The literature has primarily analyzed plant and equipment data from the Annual Survey of Manufactures (see, most notably, Cooper and Haltiwanger (2006) and Hsieh and Klenow (2009)) and accounting data for publicly-traded firms (see, most notably, David and Venkateswaran (2019) and David et al. (2022)). The dispersion estimates in these studies are typically larger than ours, generally ranging between 60 and 100 percent, depending on the time frame, firm sample, and type of capital.[24] Our findings are in line with the view that significant dispersion in the marginal product of capital arises from the nature of business capital and from the technology for producing and reallocating it across firms.

[23] In the divisible variant, only the seller's action set changes: a seller with stock k may sell any quantity $k' \leq k$ on the capital grid, rather than the entire stock, and buyers choose quantities as in the baseline. Prices form a per-quantity schedule that clears the market quantity by quantity, and the transaction cost c_T remains a fixed cost per executed transaction, paid by the buyer. Meetings, their arrival rate, and all other elements of the model are unchanged.

[24] In their study of plant and equipment in U.S. manufacturing plants, Asker et al. (2014) claim that adjustment costs are sufficient to account for the observed dispersion in marginal products of capital. We view our contribution as providing a theory-guided counterpart for the question posed by Asker et al. (2014) in the case of private business capital adjustment.

7.2 Margins of Capital Adjustment

Our parsimonious investment and trading technologies deliver all major types of investment flows documented in the literature on physical capital: large negative adjustments through sales, small to moderate positive adjustments through incremental investments, and large positive adjustments through purchases.[25] For the type of business capital we study, most of these margins cannot be observed directly: the capital is unobserved except when a business is sold. We can ask, however, whether the model's implied adjustment behavior is comparable to the plant-level evidence on physical capital, for which Census-based studies provide a detailed accounting of each margin.

For the sake of comparability, we measure investment flows in our simulated businesses the way existing studies measure plants. We simulate a panel of businesses from the baseline economy and record only the model data that would be required by the U.S. Census questionnaire: investment expenditures, purchases of capital, and the retirement of the book stock when the business is sold. Measured investment in a year is $I^{\text{book}} = \int i \, \mathrm{d}t + k^{\text{buy}} - \text{RET}$, where the integral is the year's internal investment in physical units, an acquisition adds the quantity of capital purchased, k^{buy}, and a sale retires the seller's entire book stock at its book value on the sale date, RET. Book capital K^{book} accumulates these flows net of depreciation at the rate δ_k by perpetual inventory, initialized at the true capital stock when a business enters the panel, and the investment rate $I^{\text{book}}/K^{\text{book}}$ divides the year's measured investment by book capital at the start of the year. Because book capital accumulates recorded flows only, the σ_k shocks to capital do not enter the measured rates, in line with Census-based measures that are built from expenditure and retirement flows.

Table 7 reports the distribution of the model's annual investment rates, with adjustment categories following Cooper and Haltiwanger (2006). There are several noteworthy features of the distribution that can be compared to Census studies. While the modal firm-year shows a modest positive adjustment, investment is concentrated in spikes, defined here as years with rates above 20 percent. The share of model investment occurring in spike years is 0.63. Relatedly, the model speaks to the lumpiness of investment.[26] Lumpiness declines sharply with age: young businesses build capital quickly, with a median investment rate of 0.33 and frequent spikes, whereas mature businesses make mostly moderate positive adjustments.

In terms of magnitudes, we find similarities between investment rates of mature businesses shown in the last column of Table 7 and those reported in Census plant-level studies. Among model businesses older than ten years, the mean investment rate is 0.14, the share of investment occurring in spike years is 0.42, and 13 percent of firm-years show a small positive adjustment,

[25] In line with the literature, we focus on the distribution of gross investment rates. The distribution of net investments, that is, changes in the capital stock, also includes small negative rates through depreciation.

[26] With convex investment costs and no fixed cost, the model generates small positive flows rather than the literal zeros that Cooper and Haltiwanger (2006) and Asker et al. (2014) attribute to fixed disruption costs.

Table 7: CAPITAL ADJUSTMENT IN THE MODEL
DISTRIBUTION OF ANNUAL INVESTMENT RATES

	ALL	AGE < 5	AGE > 10
Median investment rate	0.15	0.33	0.11
Small positive ($0 \leq I^{\text{book}}/K^{\text{book}} < 0.05$)	0.10	0.07	0.13
Moderate positive ($0.05 \leq I^{\text{book}}/K^{\text{book}} < 0.20$)	0.50	0.19	0.70
Positive spikes ($I^{\text{book}}/K^{\text{book}} \geq 0.20$)	0.35	0.69	0.13
Negative ($I^{\text{book}}/K^{\text{book}} \leq -0.05$)	0.05	0.05	0.04
Share of investment in spike years	0.63	0.87	0.42

Notes: Investment rates $I^{\text{book}}/K^{\text{book}}$ are computed from a simulated panel of businesses in the baseline economy (57,000 firm-years, of which 19,560 have age less than 5 and 24,885 have age greater than 10); the construction is described in the text.

with rates between zero and 5 percent. In their study of large manufacturing plants, Cooper and Haltiwanger (2006) report a mean investment rate of 0.12, roughly half of investment occurring in spike years, and an inaction rate (rates below 1 percent in absolute value) of 8 percent. The fact that there is an age gradient in the model's lumpiness pattern mirrors the size gradient that Doms and Dunne (1998) find across plants.

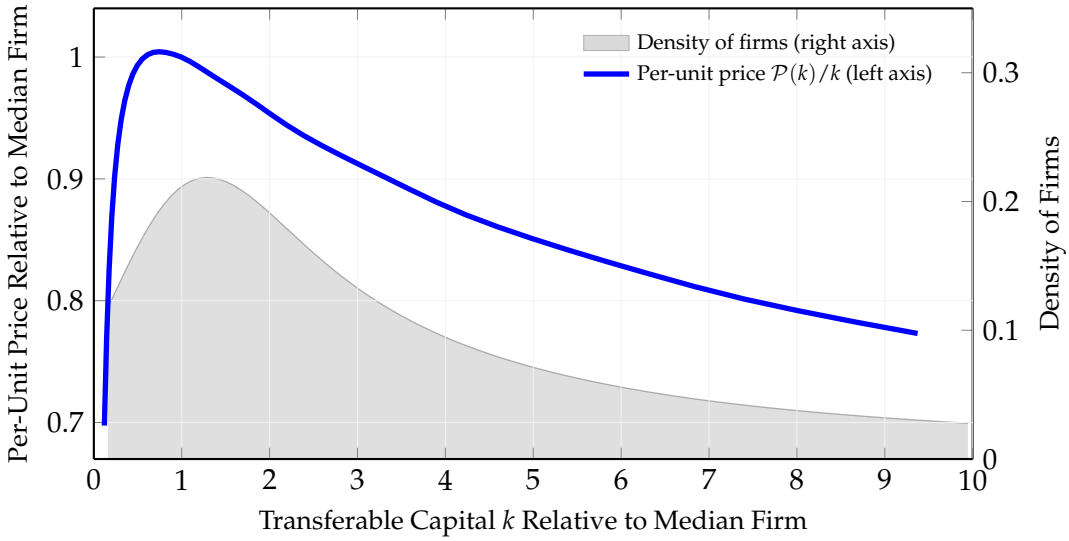
If we compare the frequency of business sales in our model simulations to Census data on plant ownership changes documented by Maksimovic and Phillips (2001), we again find similarities. Model sales occur in 4.8 percent of firm-years. This is in line with U.S. manufacturing plant-level evidence that shows an average reallocation rate of 3.6 percent in recessionary years and 6.2 percent in expansionary years. As with sales on IRS Form 8594, the relevant transfer is a group of assets that makes up a trade or business to either the buyer or seller.

7.3 Dispersion in Measures of and Returns to Business Wealth

Our model distinguishes between two concepts of business wealth. The first is the present discounted value of payments to a current owner, $V(x)$, which captures returns to transferable capital k and non-transferable capital z, as well as the option value of being a worker. We compare this measure to the equity value of a publicly traded corporation. The second measure of business wealth is often reported in surveys of consumer finances that ask respondents to estimate the price of their business if it were sold today. This measure in our model is the price of transferable capital, $\mathcal{P}(k(x))$. We use these valuation concepts to estimate variation in measures of business returns and transferable shares.

In the first column of Table 8, we report the *income yield*, a common measure of the return to

Figure 4: PER-UNIT PRICES ACROSS THE FIRM SIZE DISTRIBUTION



Notes: The solid line (left axis) is the equilibrium per-unit price of capital, $\mathcal{P}(k)/k$, in the baseline economy, relative to the per-unit price of the median firm. The shaded region (right axis) is the density of firms over transferable capital k relative to the median firm. The window covers firms between one-tenth and ten times the median size.

Table 8: PREDICTED INCOME YIELDS AND TRANSFERABLE SHARES

STATISTIC		INCOME YIELD	TRANSFERABLE SHARE
Percentiles:	5th	2.9	1.8
	10th	3.8	3.7
	25th	5.6	7.3
	50th	7.7	12.5
	75th	10.2	20.2
	90th	12.9	30.3
	95th	15.0	37.2
Average		8.2	15.0
Aggregate		11.6	18.5

Notes: The income yield is the ratio of owner income, $y(x) - wn(x) - rb(x) - c(i(x))$, to business value $V(x)$. The transferable share is the ratio of the transferable value $\mathcal{P}(k(x))$ to the total value $V(x)$. Statistics related to the distribution are reported as well as the ratios of economy-wide aggregates.

business that is given by the ratio of owner income $(1-\beta-\theta)y(x) - c(i(x))$ to business value $V(x)$. As in the case of marginal products of capital, we find significant heterogeneity in income yields, with estimates ranging from 2.9 percent at the 5th percentile of the distribution to 15.0 percent at the 95th. Comparing these results to U.S. publicly-traded companies, we find similar median and mean yields, but our estimates indicate that there is much less dispersion in private business yields than in those of publicly-traded firms. For example, Bhandari et al. (2020) compute yields ranging from -5.3 percent at the 25th to 10.4 percent at the 75th for publicly-traded businesses in the CRSP-Compustat database.[27]

The second column of Table 8 shows the *transferable share*, $\mathcal{P}(k(x))/V(x)$. As with income yields, we find significant heterogeneity in shares. At the 5th percentile, the value of transferable capital is equal to 1.8 percent times the total business value, and at the 95th percentile, the ratio is 37.2 percent. If we compute the equal- or value-weighted shares, we find 15.0 percent and 18.5 percent, respectively. Such dispersion originates from two sources. First and foremost, owners' productivity and capital are typically mismatched due to indivisibility and time to trade. In addition, the equilibrium per-unit price $\mathcal{P}(k)/k$ features a hump-shaped pattern (Figure 4). Among the smallest businesses, it rises steeply as k grows larger, since the fixed transaction cost is spread over more units of capital. The schedule peaks just below the median size. For larger firms, the per-unit price is decreasing, reflecting the discount that sellers of large, indivisible capital stocks must offer to buyers who can absorb them.

Some studies impute business wealth by capitalizing incomes (see, for example, Piketty et al. (2018)). The Flow of Funds, in contrast, imputes the value of closely-held businesses by scaling their SOI book values or revenues using publicly-traded "comparables" and applying a 25 percent discount for illiquidity. Neither approach is well suited to our context since our model generates substantial heterogeneity in income yields and in the measures of transferable shares as shown in Table 8. In contrast, our model-generated valuations can be consistently aggregated to impute measures of total private business wealth and we next put them in context by comparing them to publicly-listed companies.

For transferable capital, the total value is given by $\int \mathcal{P}(k(x))\phi(x)dx$, which we estimate to be 0.48 times private business output. It follows from the aggregate value in Table 8 that the total private business wealth, $\int V(x)\phi(x)dx$, is estimated at 2.59 times private business output. To compare this estimate to publicly-listed companies, we construct the market value of listed firms using the CRSP-Compustat database.[28] To measure the corresponding value added of these firms,

[27] Fagereng et al. (2020) and Boar et al. (2022) attempt to measure returns to private businesses using firm-level capital stocks available in their datasets. These stocks are book values and do not include the self-created intangible assets that constitute most of the value of the firms.

[28] The sample is U.S.-incorporated firms with consolidated, standard, industrial-format accounts reported in U.S. dollars, one observation per firm and fiscal year. Market value is the reported total market value of equity, with fiscal-year-end price times shares outstanding used when the former is missing. Each firm's value added is imputed as its sales times the value-added to gross-output ratio of its BEA industry (mapped from the firm's two-digit NAICS code) in

we start from their reported sales and apply industry- and year-specific ratios of value added to gross output from the BEA. This procedure imputes a BEA-consistent notion of value added for the publicly-listed sector. Comparing equity market value to this BEA-adjusted value added, we find that the ratio ranges between 1.64 in 2008 and 4.84 in 2021, with an average over the 2000–2024 period of 2.89, only slightly higher than our estimate for private businesses.

The analysis of capital trade and valuation provides key inputs to the active public debate on how to tax private business income, wealth, and capital gains. We turn to this debate next.

8 Tax Policy Analysis

In this section, we introduce taxes on labor income, business income, realized capital gains, and business wealth. We use the U.S. tax system as a baseline and evaluate several revenue-neutral reforms.

We proceed as follows. We first articulate a *benchmark principle* for taxing private business: a configuration of taxes that raises a given revenue without distorting investment, trading, entry, and exit decisions. We show how this benchmark changes in the presence of the type of business capital studied in this paper. We use the benchmark to identify the margins that the U.S. system distorts and quantify these distortions by simulating three revenue-neutral reforms: repealing the capital gains tax, equalizing the tax rates on incomes and gains, and replacing the capital gains tax with a tax on the capital stock. We close by showing that the model-implied elasticity of capital gains realizations with respect to the net-of-tax rate is in line with recent empirical estimates.

8.1 A Benchmark Principle

The business capital we study has properties that are different from those assumed in the standard framework of Hall and Jorgenson (1967) that typically guides capital taxation, and these properties lead to different taxation principles (see also Auerbach 1983, and the textbook treatment in Atkinson and Stiglitz 2015). The capital is self-created and indivisible, produced with convex costs, and reallocated through infrequent bilateral trades. With these properties, the user-cost-of-capital apparatus no longer provides the relevant reference for how capital income should be taxed. We resort to first principles using the production-efficiency benchmark of Diamond and Mirrlees (1971) and derive the implications for the taxation of private businesses.

The benchmark principle for private business tax design starts with the fact that private businesses generate cash flows through four activities: production, internal investment, transfers of

that year, using the BEA's annual value added and gross output by industry tables. The reported ratio is a ratio of aggregates: total equity market value divided by total imputed value added, with both totals summed over the same set of firm-years, namely those with a valid industry mapping.

capital, and business creation. A uniform cash-flow tax must treat the net returns from all four activities symmetrically in order to avoid distorting any marginal decision. In this ideal tax system the values of owners and workers scale equally by one minus the tax rate. Critically, investment, trading, entry, and exit decisions all coincide with those of the same economy without taxes. We next discuss the implementation issues associated with this principle and how the U.S. system departs from it.

The special case with no self-created intangible capital ($\alpha = 0$) clarifies how this principle relates to the standard cost-of-capital analysis. In this case, the only business capital is rentable fixed assets, b, owned by a mutual fund. The economic environment resembles the neoclassical model of Hall and Jorgenson (1967) with linear tax rates on wage income, business income, and the gains from the sale of capital. Uniform cash-flow taxation requires that (*a*) labor and business income be taxed at the same rate, so that occupational choice is undistorted; (*b*) investment be fully expensed (meaning that the full user cost $\rho + \delta_b$ per unit of capital services is deductible); and (*c*) sales of the mutual fund shares not be taxed. The U.S. system conforms with (*a*) for pass-through entities, whose income for the most part is taxed at ordinary income tax rates. It departs from (*b*), since depreciation δ_b rather than the full user cost is deductible, and from (*c*), since capital gains are taxed on sale price net of purchase basis upon realization. These departures depress investment relative to the optimal design.

The characteristics of business capital k in our model change the analysis in several ways. Because k is self-created, this type of capital carries no cost basis. Moreover, capital is produced with a convex cost technology that generates taxable income. The fact that business capital is indivisible and takes time to trade also implies that there is dispersion in the marginal product of this capital, unlike the fixed asset b. These features matter for tax design and change what uniform cash-flow taxation requires. Full expensing must now cover both purchases of capital from other owners *and* the owner's effort cost of internal investment. Realized capital gains from selling self-created capital—which are a part of the returns from intangible capital production—must be taxed at the same rate as other business income. The U.S. system departs on both counts. Purchased intangibles are amortized rather than expensed. More importantly, the effort cost of internally creating intangible capital is not a market transaction: no tax authority can observe or verify it, so it cannot be expensed by any feasible system—a departure that is inherent in the technology of capital formation rather than a choice of the tax code. Capital gains are taxed at preferential rates that are well below the income tax rates despite the fact that they are the income generated by the capital-production technology. The first departure from full expensing depresses investment and locks capital in with its current owners, discouraging the sales that reallocate capital toward more productive users. The second departure from equating the three tax rates distorts entry, since the option to eventually sell the business is part of the value of starting one.

Table 9: U.S. TAX SYSTEM AND REVENUE-NEUTRAL REFORMS

STATISTIC	U.S. TAX SYSTEM	REPEAL CAPITAL GAINS	EQUALIZE TAX RATES	TAX IMPUTED WEALTH
Percent change in:				
Fraction of capital traded	−47.1	−4.3	−62.1	−2.8
Dispersion in MPK	4.5	0.7	9.4	0.7
Average capital stock	−6.8	−8.0	−7.0	−8.1
Total output	−2.1	−1.4	−2.8	−1.7
Wage rate	−2.7	−1.4	−3.4	−2.4
Mass of owners	−1.9	−0.1	−2.1	−2.3
Average unit price	−27.2	−32.6	−19.8	−35.0
Deadweight loss	0.052	0.031	0.076	0.042

Notes: All columns raise the same revenue as the U.S. tax system. Entries are percent changes relative to the ideal tax scheme that leaves all decisions undistorted. The last row reports deadweight loss per dollar of revenue, defined as $-\left(1+\frac{\Delta C}{R}\right)$, where ΔC is the change in aggregate consumption (net of owners' effort costs) relative to the untaxed economy and R is the revenue raised. Tax revenue is not returned to households, so an undistorted system lowers consumption by exactly R. A positive entry is the consumption loss in excess of the revenue collected. The fraction of capital traded is the share of transferable capital reallocated through trade. Dispersion in MPK is the standard deviation of log marginal products. Average capital stock is the mean of $k(x)$ across owners. Total output is aggregate output, and wage rate is the equilibrium wage. The mass of owners is the equilibrium measure of business owners. Average unit price is the transaction-weighted mean of sale price per unit of capital, $\mathcal{P}(k)/k$.

8.2 Analysis of U.S. Tax System and Revenue-Neutral Reforms

We quantify the distortions in the U.S. tax system and then study several reforms that change the taxation of capital.

Reforms within the U.S. system. The U.S. system in our model has three tax instruments. Workers pay a tax τ_w on wage earnings. Owners pay a tax τ_b on production revenues net of the wage bill and expenditures on rented fixed assets. Sellers pay a capital gains tax τ_c on the sale price of the business. The buyer pays the intermediation fee c_T, which is neither deductible from the business-income base nor added to any cost basis.[29]

For the U.S. baseline, we set the tax rates on private business and labor income to 35 percent

[29] Our treatment departs from the U.S. tax code in two ways: we do not allow the cost basis of purchased intangibles to be deducted from the capital gains tax base, and we do not allow firms to deduct depreciation of purchased capital from their income tax base. These simplifications keep the state space of the model two-dimensional and close to the setup in Section 3. In Appendix C, we augment the model to allow for deductions of purchased capital and depreciation, maintaining our adopted convention on c_T. The results are quantitatively similar and the ranking of the reforms is unchanged, largely because most sales in our baseline economy are of self-created capital, which carries a zero basis.

and the tax rate on realized capital gains to 20 percent. We consider two reforms of tax rates, while retaining the U.S. tax bases and expensing rules. The first eliminates the capital gains tax and raises the income tax rates to replace the lost revenue. The second eliminates the preferential treatment of capital gains by setting all three tax rates equal.

The first three columns of Table 9 report the results. Each system raises the same revenue as the U.S. baseline, which has a revenue-to-output ratio of 23.2 percent. The first column sets $\tau_b = \tau_w = 35$ percent and $\tau_c = 20$ percent. The second sets $\tau_c = 0$ and raises τ_b and τ_w to 35.5 percent. The third sets all three rates to slightly less than 35 percent. Entries are percent changes relative to the ideal tax system. The last row reports deadweight loss per dollar of revenue, defined as $-\left(1 + \frac{\Delta C}{R}\right)$, where ΔC is the change in aggregate consumption (net of owners' effort costs) relative to the untaxed economy and R is the revenue raised. A positive value is the consumption loss in excess of the revenue collected.[30] The fourth column, which introduces a different tax base, is discussed separately below.

The main distortion from the capital gains tax is a reduction in trade. Under the U.S. system, the fraction of capital traded is 47.1 percent lower than under the ideal tax system. Equalizing the capital gains and income tax rates increases this shortfall to 62.1 percent. In both cases, less trade increases the dispersion in marginal products, consistent with the comparative statics in Figure 3. Eliminating the capital gains tax reduces both distortions by roughly an order of magnitude.

By contrast, neither reform removes the investment distortion. Internal investment is not fully expensed, so the business income tax continues to depress capital accumulation. Moreover, replacing the lost capital gains revenue requires a slightly higher income tax rate. As a result, the average capital stock falls by a similar amount across tax regimes. The reallocation gains dominate for output and wages. Under the U.S. system, output and the wage rate are 2.1 and 2.7 percent below their values in the nondistortionary ideal tax system, respectively. Both shortfalls are smaller when τ_c is eliminated. Equalizing the tax rates instead lowers output and wages even further due to the reduction in trade.

The capital gains tax also affects entry. It locks in marginal incumbents, which delays exit, but also lowers the value of a future sale, which discourages entry. The second effect dominates. Under the U.S. system, the mass of owners is 1.9 percent lower. Eliminating τ_c nearly eliminates this shortfall, while equalizing the rates makes it slightly larger.

Pre-tax transaction prices are lower in all three economies due to the productivity losses induced by income taxation without investment expensing. However, a capital gains tax restricts the supply of businesses offered for sale and raises the pre-tax prices needed to compensate sellers for the tax bill they incur. Thus, even though equalizing the rates produces the largest decline

[30] The aggregate C is consumption-equivalent welfare: output net of fixed-asset rental payments, owners' effort costs of investment, transaction costs, and tax revenue. Revenue is not returned to households. The ideal tax system does not change allocations. It therefore has $\Delta C = -R$ and zero deadweight loss.

in trade, average unit price in that regime is only 19.8 percent below the optimum. Prices fall monotonically when the capital gains tax is set at the current preferential level or is eliminated entirely.

The welfare results give a clear ranking. The U.S. system has a deadweight loss of 5.2 cents per dollar of revenue. The benchmark principle calls for equal tax rates, but only when investment and capital purchases are fully expensed. Under the U.S. expensing rules, raising τ_c adds a trade distortion without removing the investment distortion. The deadweight loss then *rises* to 7.6 cents. Among reforms that retain the existing U.S. tax bases, a stark deviation from the benchmark principle emerges: eliminating the capital gains tax is preferred to removing its preferential treatment, delivering a deadweight loss of only 3.1 cents per dollar of revenue.

Taxing business wealth. The first two reforms change tax rates while taking the realization-based architecture of the U.S. system as given: gains are taxed only when the asset is sold. Gains that accrue while the asset is held go untaxed, and owners can defer the tax by delaying realization. These features have motivated proposals to tax unrealized gains directly, or to tax gains before realization indirectly through an annual wealth tax.[31] There is also an efficiency case for taxing wealth, emphasized by Guvenen et al. (2023). They study an economy in which returns to capital are heterogeneous (owners differ in entrepreneurial ability) and financial frictions keep capital from flowing to its most productive users. Relative to a tax on capital income, a wealth tax places a larger burden on owners who own financial wealth but earn low returns on their businesses. It therefore shifts capital from low-productivity toward high-productivity owners, raising aggregate productivity.

We revisit this argument in the context of private business capital. Our model also generates substantial dispersion in returns to business wealth as shown in Table 8, but the source of the dispersion is not financing constraints. In the fourth column of Table 9, we introduce a tax τ_k paid each period on the imputed transferable value of the business, $\mathcal{P}(k(x))$.[32] We set τ_k to 1.38 percent and eliminate the realized capital gains tax τ_c, so that the reform raises the same revenue as the other systems. Thus, our main comparison is with the economy described in the second column, where instead income taxes are raised to maintain constant revenues.

The wealth tax does not directly discourage trade because the liability does not depend on whether the business is sold. The decline in the fraction of capital traded and the increase in marginal-product dispersion are therefore similar in these two economies, and roughly an order of magnitude smaller than under the U.S. system or the equal-tax reform. Under a tax on imputed

[31] See Saez and Zucman (2019), Scheuer and Slemrod (2021), and Azevedo et al. (2025). A wealth tax is not the same as a tax on accrued gains: it applies to the level of wealth rather than to the change in value, and it provides no refund when the value of the asset falls. The two taxes nevertheless share the feature that liabilities arise before the business is sold.

[32] This requires the tax authority to assess the transferable value of unsold businesses—a hurdle that by itself could make such proposals impractical. We abstract from such concerns here.

wealth, these gains come at the cost of lower business entry. Taxing the stock each period lowers the return to creating business capital and the value of starting a business. The mass of owners falls by 2.3 percent, the largest decline. Compared with business income, a tax on $\mathcal{P}(k(x))$ falls proportionally more on less productive, more elastic owners. Output and wages then fall by more, delivering a higher deadweight loss.

Unlike in Guvenen et al. (2023), the dispersion in returns in our model is not evidence of an inefficient allocation, but it arises because business capital is indivisible and takes time to reallocate through bilateral trade. Such properties of capital are unaffected by a wealth tax, and so is the source of return dispersion. Moreover, in our model, owners' wealth is embodied in the businesses they start and grow through investments and trade. The main effect of a wealth tax is then to reduce incentives to create business wealth, primarily by discouraging entry. We conclude that replacing the capital gains tax with τ_k avoids the lock-in effect, but it generates a higher deadweight loss than raising the income tax rates in this calibrated, revenue-neutral comparison.

8.3 The Elasticity of Capital Gains Realizations

Our policy results depend on the response of business transfers to the capital-gains tax. In this section, we compare the model-implied realization elasticity with recent empirical estimates with the caveat that available estimates cover all sources of capital gains and not exclusively those from private businesses.

Starting from the U.S. system, we raise τ_c and compute the elasticity of realizations with respect to the net-of-tax rate,

$$\varepsilon^R \;=\; \frac{d\log(\text{capital gain realizations})}{d\log(1-\tau_c)},$$

where realizations are capital gains collected in the stationary equilibrium. The model implies ε^R of around 1.41.

We express all estimates as net-of-tax elasticities to make them directly comparable.[33] Using state-level variation in capital gains tax rates, Agersnap and Zidar (2021) estimate a net-of-tax elasticity between 1.0 and 1.6 in the specification that controls for contemporaneous changes in other state taxes—the relevant comparison to our experiment, which holds τ_w and τ_b fixed. Bakija and Gentry (2014) imply a net-of-tax elasticity of roughly 1.2 to 2.6, and the values used by the Joint Committee on Taxation and the U.S. Treasury imply 2.8 and 4.0, respectively. The model-implied elasticity is therefore near the lower end of the range of empirical estimates and is closest

[33]Studies report capital gains elasticities on two bases. Agersnap and Zidar (2021) report the net-of-tax elasticity $\varepsilon^R = d\log(\text{realizations})/d\log(1-\tau_c)$ directly. Others report the elasticity with respect to the tax rate, $\varepsilon^\tau = d\log(\text{realizations})/d\log\tau_c$, which we convert using $\varepsilon^R = -\varepsilon^\tau(1-\tau_c)/\tau_c$ at the U.S. capital gains rate $\tau_c = 0.20$, so that $\varepsilon^R = -4\,\varepsilon^\tau$. The net-of-tax figures in the text come from the following originals: Bakija and Gentry (2014) report ε^τ of -0.3 to -0.66; the Joint Committee on Taxation reports $\varepsilon^\tau = -0.7$; and the U.S. Treasury reports $\varepsilon^\tau = -1$. Sarin et al. (2022) use these elasticities when making the case to increase U.S. tax rates on capital gains, but focus only on the revenue-raising potential of such a reform.

to the estimates in Agersnap and Zidar (2021).

9 Conclusion

We develop a theory of private-firm dynamics in which owner-created intangible capital (customer bases, trademarks, and going-concern value) plays a central role. We use the theory to study firm dynamics, business wealth, and business taxation. With parameters calibrated to IRS administrative tax data, the model predicts substantial dispersion in marginal products of capital, returns to business wealth, and transferable capital shares. Comparisons of the U.S. tax system with alternative systems for taxing business income and wealth reveal a clear ranking: income taxation is preferred to wealth or capital gains taxation.

We make simplifying assumptions to keep the theory and measurement transparent. We view the formation and valuation of specific components of business capital, the industrial organization consequences of business sales, and the response of business formation to taxes as particularly fruitful areas for future research on private businesses.

Our analysis focuses on the United States because of the rich data available on business transfers, but we doubt that U.S. lawyers, doctors, contractors, and other private business owners are unique in their ability to build value in their businesses. Much as existing models of firm dynamics have shaped the study of productivity and capital allocation with a focus on manufacturing, our framework, augmented with features relevant for developing economies, offers a natural starting point for a broader analysis of the role of private businesses in economic development.

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Appendix

For Theorem 1, Corollaries 1 and 2, and Theorem 2, we assume a discrete type space, $\mathcal{X} = \{x_1, ..., x_N\}$.[34] We do so to keep notation simple, but the results extend naturally to a continuum of types with appropriate measure-theoretic arguments.

A Proofs of Theorem 1 and Corollaries 1 and 2

Proof of Theorem 1. Consider problem *P1*,

$$\begin{aligned} \max_{\gamma \geq 0} \sum_{x,\tilde{x}} S(x,\tilde{x})\gamma(x,\tilde{x}) \\ \text{s.t.} \sum_{\tilde{x}} \gamma(x,\tilde{x}) \leq \phi(x)/2 \\ \sum_{x} \gamma(x,\tilde{x}) \leq \phi(\tilde{x})/2, \end{aligned}$$

and attach multipliers $\mu^a, \mu^b \geq 0$ to its two constraints. The Lagrangian is

$$L(\gamma, \mu^a, \mu^b) = \sum_{x,\tilde{x}} S(x,\tilde{x})\gamma(x,\tilde{x}) + \sum_{x} \mu^a(x)\Big[\phi(x)/2 - \sum_{\tilde{x}} \gamma(x,\tilde{x})\Big] + \sum_{\tilde{x}} \mu^b(\tilde{x})\Big[\phi(\tilde{x})/2 - \sum_{x} \gamma(x,\tilde{x})\Big].$$

The dual problem is $\min_{\mu^a,\mu^b \geq 0} \max_{\gamma \geq 0} L(\gamma, \mu^a, \mu^b)$; carrying out the inner maximization yields the dual of *P1*,

$$\begin{aligned} \min_{\mu^a,\mu^b \geq 0} \sum_{x} \mu^a(x)\phi(x)/2 + \sum_{\tilde{x}} \mu^b(\tilde{x})\phi(\tilde{x})/2 \\ \text{s.t. } \mu^a(x) + \mu^b(\tilde{x}) \geq S(x,\tilde{x}). \end{aligned}$$

By strong duality for finite linear programs, the primal and dual values are equal and both problems are attained. Since S is symmetric, if (μ^a, μ^b) is an optimal dual pair, so is the swapped pair (μ^b, μ^a): it satisfies every constraint by symmetry of S, and the objective is unchanged because both sides carry the weights $\phi/2$. The average of the two pairs is feasible, by averaging the constraints at $(x,\tilde{x})$ and $(\tilde{x},x)$, and optimal. Hence an optimal dual with $\mu^a = \mu^b \equiv \mu$ exists; we work with this symmetric solution throughout and label its common value μ. We proceed in two steps.

First, within a match, the surplus S selects the trade that assigns the combined capital to the owner who values it more, so the post-trade stocks satisfy (1) by construction. Post the price (15) for *every* ordered pair $(x,\tilde{x})$, that is, for the transaction in which x buys the stock of $\tilde{x}$, and set $p^s(\tilde{x},x) = p^b(x,\tilde{x})$, so the price-consistency requirement (10) holds for all pairs by construction. Under this posting, the buyer's payoff from any transaction is exactly $\mu(x)$, and the seller's payoff

[34] On a finite grid, the post-trade capital $k(x) + k(\tilde{x})$ need not itself be a grid point. None of the arguments in this appendix depend on the additive form of the post-trade state: they apply verbatim with an arbitrary post-trade map $T^b(x,\tilde{x}) \in \mathcal{X}$ in place of $(z(x), k(x) + k(\tilde{x}))$ in the surplus, the trade operator, and the price formulas. In the numerical implementation, an off-grid sum is assigned to the two bracketing grid points by a lottery that preserves the expected post-trade capital.

is the joint gain of that direction minus the buyer's multiplier, which is at most $\mu(\tilde{x})$ because the joint gain is at most $S(x,\tilde{x}) \leq \mu(x) + \mu(\tilde{x})$ by dual feasibility. For matches that form, $\gamma(x,\tilde{x}) > 0$, complementary slackness gives $\mu(x) + \mu(\tilde{x}) = S(x,\tilde{x})$, and the executed direction attains the maximum in S by the construction of the trade direction; substituting shows the seller's payoff is exactly $\mu(\tilde{x})$ and the posted price coincides with (16) evaluated at the seller's state. Complementary slackness with respect to the constraints of *P1* also implies that $\mu(x) = 0$ for any state x whose constraint is slack, that is, for owners who leave the market unmatched with positive probability. The trading rates assign the total in (18) to the role selected by the surplus maximization; for same-state pairs the two roles split the total equally. So constructed, the rates satisfy the consistency condition (11) and are positive only for matches that form; summing the constraints of *P1* gives $\sum_{\tilde{x}}[\lambda^b(x,\tilde{x}) + \lambda^s(x,\tilde{x})] \leq 1$, with slack exactly for owners who leave the market unmatched with positive probability. At states with $\phi(x) = 0$ the constraints of *P1* force $\gamma(x,\cdot) = \gamma(\cdot,x) = 0$, and the rates may be set to any solution of (3); consistency holds because all transaction masses involving such states are zero.

Second, we verify condition (17). Under the posted prices, every buying option pays $v^b(x,\tilde{x}) - V(x) = \mu(x)$ and every selling option pays $v^s(x,\tilde{x}) - V(x) \leq \mu(x)$, with equality on executed trades. Since $\mu \geq 0$ and the option not to trade pays zero, the maximum in (3) equals $\mu(x)$ at *every* $x \in \mathcal{X}$, including states that never trade. The trading rates deliver exactly that maximum: they place probability only on trades that form, each paying $\mu(x)$, and an owner with $\mu(x) > 0$ trades with probability one, since her constraints in *P1* bind; an owner with $\mu(x) = 0$ is indifferent over trading probabilities. Hence $V_{trade}(x) = \mu(x)$, which completes the proof of part (i). Q.E.D.

Before proving part (ii), we record an identity used there and again in the proof of Theorem 2. Fix V and a measure ξ, and let λ be trading rates satisfying the probability constraint in (3) and the consistency condition (11) at ξ, with capital allocations satisfying (1). The plan's joint continuation-value gain net of transaction costs, per market-access opportunity, is

$$T_V(\lambda,\xi) \equiv \sum_{x,\tilde{x}} \xi(x)\Big[\lambda^b(x,\tilde{x})\big(V(z(x),k(x)+k(\tilde{x})) - V(x) - c_T\big) + \lambda^s(x,\tilde{x})\big(V(z(x),0) - V(x)\big)\Big]. \tag{19}$$

Lemma 1 (Trade-value identity)**.** *Let $D^b(x,\tilde{x}) = V(z(x),k(x)+k(\tilde{x})) + V(z(\tilde{x}),0) - V(x) - V(\tilde{x}) - c_T$ and $D^s(x,\tilde{x}) = D^b(\tilde{x},x)$ denote the joint gains from the two directions of trade. Then*

$$T_V(\lambda,\xi) = \frac{1}{2}\sum_{x,\tilde{x}} \xi(x)\big[\lambda^b(x,\tilde{x})D^b(x,\tilde{x}) + \lambda^s(x,\tilde{x})D^s(x,\tilde{x})\big]. \tag{20}$$

Moreover, the induced assignment $\gamma_\lambda(x,\tilde{x}) = \frac{\xi(x)}{2}[\lambda^b(x,\tilde{x}) + \lambda^s(x,\tilde{x})]$ is symmetric and feasible for P1 *at ξ, and since $D^j(x,\tilde{x}) \leq S(x,\tilde{x})$ for both directions,*

$$T_V(\lambda,\xi) \leq \sum_{x,\tilde{x}} \gamma_\lambda(x,\tilde{x})S(x,\tilde{x}) \leq Q(\xi,V). \tag{21}$$

Proof. Consistency pairs each buyer flow with the corresponding seller flow: $\lambda^s(\tilde{x}, x)\xi(\tilde{x}) = \lambda^b(x, \tilde{x})\xi(x)$. Re-indexing the seller sum in (19) by the counterparty and splitting each transaction's total equally across its two appearances gives (20): each physical transaction enters the right-hand side twice, once through the buyer's rate and once through the seller's, each time with half of its joint gain. Symmetry of γ_λ follows from consistency applied at both orderings, the row constraint of *P1* from the probability constraint, and the column constraint from symmetry. The bound (21) follows from $D^j \leq S$, the symmetry of S, and feasibility of γ_λ. Q.E.D.

Proof of Theorem 1, part (ii). Let (p^j, k^j, λ^j) be admissible trading arrangements given (ϕ, V), write $\mu(x) \equiv V_{trade}(x)$, and let $\gamma(x, \tilde{x}) = \frac{1}{2}\phi(x)[\lambda^b(x, \tilde{x}) + \lambda^s(x, \tilde{x})]$ be the induced assignment from the theorem. Write $\pi^j(x, \tilde{x}) \equiv v^j(x, \tilde{x}) - V(x)$ for the private gain from a contract. Since an owner can put all trading probability on any one contract or can leave the market without trading, attainment of the maximum in (3) implies

$$\pi^j(x, \tilde{x}) \leq \mu(x) \quad \text{for every } (j, x, \tilde{x}), \tag{22}$$

and $\mu(x) \geq 0$. Price consistency (10) implies that, if x buys from $\tilde{x}$, the two owners' gains sum to the joint gain of that direction: $\pi^b(x, \tilde{x}) + \pi^s(\tilde{x}, x) = D^b(x, \tilde{x})$. Applying (22) to this direction and to the reverse direction gives

$$S(x, \tilde{x}) \leq \mu(x) + \mu(\tilde{x}), \tag{23}$$

so (μ, μ) is dual feasible.

If a contract is executed with positive mass, both parties place positive probability on it, so it belongs to the support of a solution attaining the maximum in (3) for each of them, and each side's private gain equals her μ; this also holds when $\mu(x) = 0$, because the option not to trade yields zero. Hence the joint gain from every executed direction equals $\mu(x) + \mu(\tilde{x})$. Since this direction is one of the two alternatives defining S, inequality (23) must bind on executed transactions, and the executed direction attains $S(x, \tilde{x})$.

By Lemma 1, γ is symmetric and feasible for *P1*. If $\mu(x) > 0$, the probability constraint in (3) binds, so $\sum_{\tilde{x}} \gamma(x, \tilde{x}) = \phi(x)/2$. It follows that

$$\sum_{x,\tilde{x}} \gamma(x, \tilde{x}) S(x, \tilde{x}) = \sum_{x,\tilde{x}} \gamma(x, \tilde{x}) \big[\mu(x) + \mu(\tilde{x})\big] = \sum_{x} \mu(x)\phi(x),$$

where the first equality uses $S = \mu(x) + \mu(\tilde{x})$ on the support of γ, and the second uses symmetry of γ, the binding constraint where $\mu(x) > 0$, and the vanishing of terms where $\mu(x) = 0$. The left side is the value of a primal-feasible assignment and the right side the value of a dual-feasible solution. Weak duality implies both are optimal. Finally, every executed direction attains S, so evaluating the grouping identity (20) on the plan gives

$$T_V(\lambda, \phi) = \sum_{x,\tilde{x}} \gamma(x, \tilde{x}) S(x, \tilde{x}) = \sum_{x} \mu(x)\phi(x) = Q(\phi, V),$$

and the plan's joint continuation-value gain net of transaction costs per market-access opportunity, $T_V(\lambda, \phi)$, equals $Q(\phi, V) = \sum_x V_{trade}(x)\phi(x)$. Q.E.D.

Proof of Corollary 1. Prices only redistribute value within a pair, so a deviating pair can jointly gain at most the surplus $S(x, \tilde{x})$. By the dual constraint and (17), $S(x, \tilde{x}) \leq \mu(x) + \mu(\tilde{x}) = V_{trade}(x) + V_{trade}(\tilde{x})$: a pair cannot jointly gain more than its members already earn in equilibrium, so no deviation can leave both owners weakly better off, with one strictly better off. Q.E.D.

Proof of Corollary 2. On executed sales, the seller's price equals (16) by Theorem 1, and the right-hand side of (16) involves no $\tilde{x}$; denote it by $p^s(x)$. It remains to show that it depends on x only through the quantity sold, $k(x)$. Consider owners with states x_1 and x_2 with $k(x_1) = k(x_2) = k$ whose sales to buyers with states $\tilde{x}_1$ and $\tilde{x}_2$, respectively, are executed. For an executed pair, complementary slackness gives $\mu(\tilde{x}_i) + \mu(x_i) = S(\tilde{x}_i, x_i)$, and the executed trade attains the maximum in the surplus, so $S(\tilde{x}_i, x_i) = V(z(\tilde{x}_i), k(\tilde{x}_i) + k) - V(\tilde{x}_i) - c_T + V(z(x_i), 0) - V(x_i)$. For the cross pair—buyer $\tilde{x}_1$ facing seller x_2—dual feasibility gives $\mu(\tilde{x}_1) + \mu(x_2) \geq S(\tilde{x}_1, x_2) \geq V(z(\tilde{x}_1), k(\tilde{x}_1) + k) - V(\tilde{x}_1) - c_T + V(z(x_2), 0) - V(x_2)$. Subtracting the equality for $i = 1$ and rearranging using (16) yields $p^s(x_2) \geq p^s(x_1)$. Exchanging the roles of x_1 and x_2 yields the reverse inequality. Hence all executed sales of the same quantity command the same price. Define $\mathcal{P}(k)$ as this common price on quantities sold with positive mass, and extend $\mathcal{P}$ arbitrarily to the remaining quantities so that its domain is K.

For the completion claim, fix any owner x who sells with positive probability and replace the prices of her unexecuted sales by $\bar{p}^s(x, \tilde{x}) = \mathcal{P}(k(x)) = V(x) + \mu(x) - V(z(x), 0)$ and $\bar{p}^b(\tilde{x}, x) = \bar{p}^s(x, \tilde{x})$ for every counterparty $\tilde{x}$. Under this completion, the seller's gain from any such sale is $V(z(x), 0) + \bar{p}^s(x, \tilde{x}) - V(x) = \mu(x)$, so her gains from trade are unchanged. A potential buyer with state $\tilde{x}$ obtains $D^b(\tilde{x}, x) - \mu(x)$, which is at most $\mu(\tilde{x})$ by dual feasibility, so no buyer strictly prefers such a purchase to her equilibrium gains. The original allocation and trading rates therefore remain privately optimal and consistent, and $p^s(x, \tilde{x}) = \mathcal{P}(k(x))$ holds for every potential counterparty of every owner who sells with positive probability. Q.E.D.

B Proof of Theorem 2

Fixed assets

For completeness, consider first the formulation in which the planner accumulates the stock of fixed assets. Let X_t denote gross investment. Then

$$\dot{B}_t = X_t - \delta_b B_t, \qquad B_t = \sum_x b_t(x)\phi_t(x), \tag{24}$$

with B_0 given. For every admissible path satisfying $\lim_{t\to\infty} e^{-\rho t} B_t = 0$, integration by parts gives

$$\begin{aligned}\int_0^\infty e^{-\rho t} X_t \, dt &= -B_0 + (\rho + \delta_b) \int_0^\infty e^{-\rho t} B_t \, dt \\ &= -B_0 + (\rho + \delta_b) \int_0^\infty e^{-\rho t} \sum_x b_t(x) \phi_t(x) \, dt.\end{aligned}$$

Consequently, conditional on B_0, subtracting investment expenditures is equivalent, up to the constant B_0, to charging the user cost $r = \rho + \delta_b$ for fixed assets. Given this we formulate the planner's problem directly in terms of the user-cost r and omit the state variable B_t from the planner's problem.

Overview of the proof

The central difficulty in establishing efficiency of the stationary recursive equilibrium is that the planner may choose any feasible path, including a nonstationary one. The proof below overcomes this difficulty in two steps. First, we construct an upper bound on the payoff of every feasible planner path using private optimality. Then we show that the stationary equilibrium attains the resulting bound when the planner starts from the stationary measure.

Notation

We build notation for various objects that will be used in the proof.

For a function h and a measure ξ, write $\langle h, \xi \rangle = \sum_x h(x)\xi(x)$, and write $S_V \equiv S$ for the surplus of Section 4.1, computed at the fixed supporting value function V. Let $\mathcal{L}^0_{i,o}$ denote the generator of the owner's state excluding trade and entry, when investment is i and the occupational-exit indicator is $o(x) \equiv \iota^{\text{out}}(x)$. Thus, in the continuous-state notation used in the text,

$$\begin{aligned}\mathcal{L}^0_{i,o} h(x) = (i(x) - \delta_k k)\partial_k h(x) + \mu_z(z)\partial_z h(x) + \frac{1}{2}\sigma_z(z)^2 \partial_{zz} h(x) + \frac{1}{2}\sigma_k^2 k^2 \partial_{kk} h(x) \\ + \delta_{\text{exo}}\big(h(z,0) - h(x)\big) - \psi_e h(x) - \psi_o o(x) h(x). \qquad (25)\end{aligned}$$

The term $-\psi_e h(x)$ corresponds to death, and occupational exit removes an owner from the measure ϕ. For an entry rule $\iota^{\text{in}} : \mathcal{X} \to \{0,1\}$, the induced entry-flow measure is

$$e_{\iota^{\text{in}}}(x) \equiv \psi_e \iota^{\text{in}}(x) g(x),$$

where $g(x)$ is the entrant probability mass at state x, the finite-state counterpart of the entrant measure $dG(x)$ in the main text. Finally, for a measure ξ, let $\Gamma_\lambda(\xi)$ denote the change in the measure of owner states induced by trade, including the arrival rate η. With δ_q denoting a unit mass at state q, its finite-state definition is

$$\begin{aligned}\Gamma_\lambda(\xi) = \eta \sum_x \xi(x) \sum_{\tilde{x}} \Big[&\lambda^b(x,\tilde{x})\big(\delta_{(z(x),k(x)+k(\tilde{x}))} - \delta_x\big) \\ &+ \lambda^s(x,\tilde{x})\big(\delta_{(z(x),0)} - \delta_x\big)\Big].\end{aligned}$$

Summing over states gives $\langle 1, \Gamma_\lambda(\xi)\rangle = 0$: trade preserves the mass of owners. The law of motion can then be written as

$$\dot{\phi} = (\mathcal{L}^0_{i,o})^* \phi + e_{\iota^{\text{in}}} + \Gamma_\lambda(\phi). \tag{26}$$

For any feasible and consistent trade plan λ at a measure ξ, the joint continuation-value gain net of transaction costs per unit of time satisfies

$$\langle V, \Gamma_\lambda(\xi)\rangle - \eta c_T \sum_{x,\tilde{x}} \lambda^b(x,\tilde{x})\xi(x) = \eta\, T_V(\lambda, \xi), \tag{27}$$

with T_V as defined in (19): pairing each transaction's change in the buyer's and the seller's continuation values, and subtracting the transaction cost once through the buyer flow, reproduces the two bracketed terms of (19). Lemma 1 provides the detailed accounting.

A trade bound

We now show that the value of any feasible trade plan is bounded by the value of a dual solution of *P1* at the current measure.

Lemma 2 (Trade bound)**.** *Let μ be any nonnegative function satisfying*

$$\mu(x) + \mu(\tilde{x}) \geq S_V(x,\tilde{x}) \quad \textit{for every } (x,\tilde{x}), \tag{28}$$

with S_V as defined above. Then, for every measure ξ and every feasible and consistent trade plan λ at that measure,

$$\langle V, \Gamma_\lambda(\xi)\rangle - \eta c_T \sum_{x,\tilde{x}} \lambda^b(x,\tilde{x})\xi(x) \leq \eta \langle \mu, \xi\rangle. \tag{29}$$

Proof. The constraints (28) do not depend on the measure, so (μ, μ) is dual feasible for *P1* at every ξ, and weak duality gives $Q(\xi, V) \leq \langle \mu, \xi\rangle$. Lemma 1 bounds the plan's value: $T_V(\lambda, \xi) \leq Q(\xi, V)$. Combining the two with (27) proves (29). □

Proof of Theorem 2

With the notation above, we now prove that the stationary recursive equilibrium is efficient—that is, it achieves $P(\phi^*)$ in problem *P2*.

Consider a stationary recursive equilibrium with measure ϕ^*, and let

$$H(x) \equiv V(x) - W, \qquad w = (\rho + \psi_e)W.$$

The stationary equilibrium path is feasible for problem *P2* by Definition 1, and the equilibrium trading arrangements are admissible given (ϕ^*, V). By Theorem 1, part (ii), $\mu(x) \equiv V_{\text{trade}}(x)$ is the common value of a symmetric optimal dual solution of *P1* at (ϕ^*, V)—in particular, μ satisfies (28), so Lemma 2 applies at every measure—and the equilibrium trade plan attains

$$\langle V, \Gamma_\lambda(\phi^*)\rangle - \eta c_T \sum_{x,\tilde{x}} \lambda^b(x,\tilde{x})\phi^*(x) = \eta \langle \mu, \phi^*\rangle. \tag{30}$$

We first use these equilibrium objects to derive an upper bound on the planner's value from an arbitrary admissible initial measure ϕ_0. We then specialize to $\phi_0 = \phi^*$ and show that the stationary equilibrium allocation attains this bound.

Subtracting $(\rho + \psi_e)W = w$ from the owner's HJB and using the definition of the generator in (25) gives

$$\rho H(x) = \max_{\substack{n,b,i \geq 0 \\ o \in \{0,1\}}} \left\{ y(x,b,n) - rb - c(i) - w(1+n) + \mathcal{L}^0_{i,o} H(x) \right\} + \eta \mu(x). \tag{31}$$

Here the maximization over o reproduces $\psi_o \{W - V(x)\}^+$.

Because of the maximization, equation (31) implies the following inequality for every admissible choice (n, b, i, o):

$$y(x,b,n) - rb - c(i) - w(1+n) + \mathcal{L}^0_{i,o} H(x) \leq \rho H(x) - \eta \mu(x). \tag{32}$$

Now consider any feasible, possibly nonstationary, planner allocation starting from an arbitrary admissible initial measure ϕ_0. Denote its instantaneous flow payoff—output net of rental payments, effort costs of investment, and transaction costs—by

$$R_t = \sum_x \left[y(x, b_t, n_t) - r b_t(x) - c(i_t(x)) \right] \phi_t(x) - \eta c_T \sum_{x,\tilde{x}} \lambda^b_t(x, \tilde{x}) \phi_t(x).$$

Multiplying (32) by $\phi_t(x)$, summing over x, and using labor feasibility, $\sum_x (1 + n_t(x)) \phi_t(x) = 1$, gives

$$\begin{aligned} R_t \leq w + \rho \langle H, \phi_t \rangle - \left\langle H, (\mathcal{L}^0_{i_t, o_t})^* \phi_t \right\rangle - \eta \langle \mu, \phi_t \rangle \\ - \eta c_T \sum_{x,\tilde{x}} \lambda^b_t(x, \tilde{x}) \phi_t(x). \end{aligned} \tag{33}$$

Because trade preserves the mass of owners, $\langle H, \Gamma_\lambda \rangle = \langle V, \Gamma_\lambda \rangle$. Lemma 2 therefore implies

$$-\eta \langle \mu, \phi_t \rangle - \eta c_T \sum_{x,\tilde{x}} \lambda^b_t(x, \tilde{x}) \phi_t(x) \leq -\langle H, \Gamma_{\lambda_t}(\phi_t) \rangle.$$

Substituting this inequality and the law of motion (26) into (33) yields

$$R_t \leq w + \rho \langle H, \phi_t \rangle - \langle H, \dot{\phi}_t \rangle + \langle H, e_{\iota^{\text{in}}_t} \rangle. \tag{34}$$

Next we show that the equilibrium entry rule maximizes the last term. In particular, define

$$E(H) \equiv \psi_e \sum_x g(x) \{H(x)\}^+.$$

We have that $\langle H, e_{\iota^{\text{in}}_t} \rangle = \psi_e \sum_x \iota^{\text{in}}_t(x) H(x) g(x)$, and because $\iota^{\text{in}}_t(x) \in \{0, 1\}$, each term is at most $\psi_e \{H(x)\}^+ g(x)$. The equilibrium rule (5) enters exactly at states with $H(x) = V(x) - W > 0$, so it attains the bound state by state; states with $H(x) = 0$ contribute zero under any rule. Thus, for every feasible entry rule, $\langle H, e_{\iota^{\text{in}}_t} \rangle \leq E(H)$ and (34) becomes

$$R_t \leq w + E(H) + \rho \langle H, \phi_t \rangle - \langle H, \dot{\phi}_t \rangle. \tag{35}$$

Integrating (35) up to an arbitrary date T gives

$$\int_0^T e^{-\rho t} R_t \, dt \leq \frac{w + E(H)}{\rho}(1 - e^{-\rho T}) + \langle H, \phi_0 \rangle - e^{-\rho T}\langle H, \phi_T \rangle. \tag{36}$$

For the global upper bound it is not necessary that the terminal term in (36) converge to zero; a one-sided bound suffices, and it follows from the constraints of *P2*. Since $V : \mathcal{X} \to \mathbb{R}_+$ and $W \in \mathbb{R}_+$ (Definition 1),

$$H(x) = V(x) - W \;\geq\; -W.$$

Moreover, ϕ_t is a nonnegative measure, so the date-t labor constraint and $n_t(x) \geq 0$ imply $\langle 1, \phi_t \rangle \leq \langle 1 + n_t, \phi_t \rangle = 1$ for all t. Consequently,

$$-e^{-\rho T}\langle H, \phi_T \rangle \;\leq\; We^{-\rho T}\langle 1, \phi_T \rangle \;\leq\; We^{-\rho T},$$

and (36) becomes

$$\int_0^T e^{-\rho t} R_t \, dt \;\leq\; \frac{w + E(H)}{\rho}\big(1 - e^{-\rho T}\big) + \langle H, \phi_0 \rangle + We^{-\rho T}.$$

Letting $T \to \infty$ (taking the lim sup of truncated payoffs if the improper integral is not defined) shows that every feasible planner allocation starting from an admissible initial measure ϕ_0 has payoff bounded above by

$$\frac{w + E(H)}{\rho} + \langle H, \phi_0 \rangle.$$

Taking the maximum over feasible allocations therefore gives

$$P(\phi_0) \leq \frac{w + E(H)}{\rho} + \langle H, \phi_0 \rangle \qquad \text{for every admissible } \phi_0. \tag{37}$$

We now specialize to $\phi_0 = \phi^*$. Evaluating (37) at $\phi_0 = \phi^*$ immediately gives one side of the desired equality:

$$P(\phi^*) \leq \frac{w + E(H)}{\rho} + \langle H, \phi^* \rangle. \tag{38}$$

For the reverse inequality, it suffices to exhibit a feasible allocation from ϕ^* whose payoff equals the right-hand side of (38). The stationary equilibrium allocation is such an allocation. It is feasible for problem *P2* by Definition 1, and starting from ϕ^* it keeps the distribution constant: $\phi_t = \phi^*$ and hence $\dot{\phi}_t = 0$ for every t. Under this allocation, all the inequalities used in deriving (37) hold with equality: owners maximize (31), the equilibrium trade plan attains (30) by Theorem 1, part (ii), and entry maximizes $\langle H, e_{t^{\text{in}}} \rangle$. Its stationary net-output flow R^* therefore satisfies

$$R^* = w + E(H) + \rho\langle H, \phi^* \rangle,$$

and its payoff is

$$\int_0^\infty e^{-\rho t} R^* \, dt = \frac{w + E(H)}{\rho} + \langle H, \phi^* \rangle.$$

Since $P(\phi^*)$ is the maximal payoff over feasible allocations,

$$\frac{w + E(H)}{\rho} + \langle H, \phi^* \rangle \leq P(\phi^*).$$

Combining the two inequalities gives $P(\phi^*) = \frac{w+E(H)}{\rho} + \langle H, \phi^* \rangle$, attained by the stationary equilibrium allocation. □

C Model with Cost Basis and Depreciation of Purchased Capital

The U.S. tax system combines two deductions that the baseline equilibrium of Section 3 abstracts from. First, each period the owner deducts depreciation of purchased assets from operating profit when computing taxable business income. Second, at the moment of a sale, the seller deducts the original cost of the capital from the realized price, so the capital-gains tax falls only on the gain over basis. By and large, both deductions are computed against a single residual quantity: the unamortized cost basis carried by the current owner. To represent the U.S. system in our environment it is therefore sufficient to enlarge the owner's state by one non-negative variable, a, that tracks this basis. This appendix describes the law of motion of a, and how it enters the value function and the gains from trade. Without taxes the basis is payoff-irrelevant and the augmented equilibrium collapses to the baseline of Section 3.

C.1 Recursive formulation

State variables. The owner's state is now $x = (z, k, a) \in \mathcal{X} \times \mathbb{R}_+$, augmenting the baseline state $(z, k) \in \mathcal{X}$ with the basis a, which derives from past purchases of k and evolves as follows.

Between trades it decays at the depreciation rate of physical capital,

$$\mathrm{d}a \;=\; -\,\delta_k\, a\, \mathrm{d}t, \tag{39}$$

which is the continuous-time analogue of the standard tax-accounting rule that amortizes a depreciable asset in proportion to its remaining undepreciated value. U.S. law instead amortizes acquired Section 197 intangibles straight-line over fifteen years. We adopt geometric decay at the rate δ_k because it keeps the basis a single state variable—no purchase vintages need to be tracked. New owners enter with no acquired basis: investment in self-created capital is not added to a, consistent with the U.S. treatment of self-created intangibles, so a records only the cost of capital purchased from another owner. When an owner buys the counterparty's capital $\tilde{k}$ at price $p^b(x, \tilde{x})$, the entire purchase price is recorded as new basis,

$$a \;\longmapsto\; a \,+\, p^b(x, \tilde{x}). \tag{40}$$

At a sale, the seller exits to $(z, 0, 0)$ so that her a is reset to 0. With these rules in place, the basis a is fully determined by the owner's history of trade and depreciation. Equilibrium prices are

positive, so the basis remains nonnegative. The transaction cost c_T is not capitalized into basis, an abstraction from the U.S. treatment of acquisition costs.

Tax deductions The income-tax deduction is applied every period. Write $\pi(x)$ for operating profit, $y(x,b,n) - rb - wn$ evaluated at the owner's optimal fixed-asset and labor choices, as in the baseline of Section 3. The τ_b base is operating profit net of the basis depreciation,

$$\text{base}_b(x) \;=\; \pi(x) \;-\; \delta_k\, a, \tag{41}$$

so each dollar of basis decay in (39) reduces the tax bill by τ_b dollars. The owner's after-tax operating flow is

$$(1-\tau_b)\,\pi(x) \;+\; \tau_b\,\delta_k\,a. \tag{42}$$

The effort cost $c(i)$ and the buyer's payment p^b are paid from after-tax resources and are not immediately deductible. The basis the payment creates is amortized over time through (41) rather than deducted at the moment of payment, a departure from the ideal tax system we describe in Section 8.

The capital-gains-tax deduction is applied at the moment of sale. At a sale of state $x = (z,k,a)$ at price $p^s(x,\tilde{x})$, the seller is taxed on the realized gain over basis,

$$\text{base}_c(x,\tilde{x}) \;=\; p^s(x,\tilde{x}) \;-\; a, \tag{43}$$

so the after-tax cash proceeds are

$$(1-\tau_c)\,p^s(x,\tilde{x}) \;+\; \tau_c\,a. \tag{44}$$

A seller who carries a larger basis pays less tax on the same sale price. Two simplifications are built into (43). The entire gain is taxed at the single rate τ_c, with no ordinary-income recapture of prior amortization and no asset-by-asset allocation of the price. The base is also linear: a seller whose price falls short of basis receives a contemporaneous credit $\tau_c\,(a - p^s)$, with no loss limits or carryforwards.

Hamilton–Jacobi–Bellman equation On the augmented state, the owner value $V(x)$ solves

$$\begin{aligned}(\rho+\psi_e)\,V(x) \;=\; \max_{n,b,i,\lambda}\Big\{ & (1-\tau_b)\big(y(x,b,n) - rb - wn\big) \;+\; \tau_b\,\delta_k\,a \;-\; c(i) \\ & +\; \partial_k V(x)\,(i-\delta_k k) \;-\; \delta_k\,a\,\partial_a V(x) \\ & +\; \mu_z(z)\,\partial_z V(x) \;+\; \tfrac{1}{2}\sigma_z(z)^2\,\partial_{zz} V(x) \;+\; \tfrac{1}{2}\sigma_k^2 k^2\,\partial_{kk} V(x) \\ & +\; \psi_o\,\{W(z) - V(x)\}^+ \;+\; \delta_{\text{exo}}\big(V(z,0,0) - V(x)\big) \;+\; \eta\,V_{trade}(x;\lambda)\Big\}.\end{aligned} \tag{45}$$

The structure mirrors the baseline HJB (2). The new elements are confined to the basis: the operating flow is taxed at τ_b with the depreciation rebate (42), the basis has its own deterministic drift

$-\delta_k a\,\partial_a V$, and the seller's side of the trade option is evaluated with the after-tax proceeds (44). Everything else—the investment first-order condition, the productivity drift $\mu_z(z)$ and diffusion $\sigma_z(z)$, the capital diffusion σ_k, the occupation option, the exogenous firm exit at rate δ_{exo}, and the trade arrival rate η—is inherited from the baseline. Two conventions complete the description. When the stock is destroyed at rate δ_{exo}, the owner moves to $(z, 0, 0)$: the remaining basis is extinguished with the capital. The worker value $W(z)$ is computed as in the baseline model with the wage flow taxed at τ_w.

The trade option itself takes the same form as in the main text but on the enlarged state. A buyer of type $x = (z, k, a)$ who meets a counterparty $\tilde{x} = (\tilde{z}, \tilde{k}, \tilde{a})$ pays $p^b(x, \tilde{x})$ and the transaction cost c_T, exits the trading stage with capital $k^b(x, \tilde{x})$, and records the purchase price as basis via (40),

$$v^b(x, \tilde{x}) \;=\; V\big(z(x),\, k^b(x, \tilde{x}),\, a + p^b(x, \tilde{x})\big) \;-\; p^b(x, \tilde{x}) \;-\; c_T.$$

A seller of type x exits to $(z, 0, 0)$ and receives the after-tax cash from (44),

$$v^s(x, \tilde{x}) \;=\; V\big(z(x),\, k^s(x, \tilde{x}),\, 0\big) \;+\; (1 - \tau_c)\, p^s(x, \tilde{x}) \;+\; \tau_c\, a.$$

The gains-from-trade $V_{trade}(x; \lambda)$ is the same as in the main text once these surpluses are substituted in. The feasibility and consistency constraints are unchanged. Two remarks are in order. First, with a basis in the state, the price is no longer a pure transfer within a trading pair: an extra dollar paid raises the buyer's future deductions through $a + p^b$, while the seller keeps $1 - \tau_c$ of it. The price-neutrality of pair surplus in the baseline therefore does not extend to the augmented model, which we use only quantitatively. Second, the pricing protocol is unchanged: prices form a per-quantity schedule that clears the market quantity by quantity, so executed prices depend only on the quantity of capital traded. Heterogeneity in basis shifts owners' trading probabilities, not the price schedule.

C.2 Law of motion

Let $\phi(z, k, a)$ denote the stationary measure on the augmented state. It evolves according to

$$\dot{\phi}(x) \;=\; \big(\mathcal{A}^*_{(i,\lambda)}\,\phi\big)(x) \;+\; \psi_e\, \iota^{\text{in}}(x)\, dG(x), \tag{46}$$

where $\mathcal{A}^*_{(i,\lambda)}$ is the conjugate operator of the baseline KFE (7) extended to the augmented state. Four elements are new. Within each (z, k) slice the basis drifts deterministically at rate $-\delta_k a$ from (39). Trade events couple (k, a) at the jump: a buyer's mass at (z, k, a) relocates to $\big(z, k^b(x, \tilde{x}), a + p^b(x, \tilde{x})\big)$ via (40), while a seller's mass relocates to $(z, 0, 0)$; exogenous firm exit, at rate δ_{exo}, likewise sends mass to $(z, 0, 0)$. Finally, entrants are placed at $a = 0$. The mass of owners m evolves as in the main text and the labor and capital market-clearing conditions are unchanged.

Table 10: U.S. TAX SYSTEM AND REVENUE-NEUTRAL REFORMS
AUGMENTED MODEL WITH DEDUCTIONS

STATISTIC	U.S. TAX SYSTEM	REPEAL CAPITAL GAINS	EQUALIZE TAX RATES	TAX IMPUTED WEALTH
Percent change in:				
Fraction of capital traded	−27.6	12.6	−46.1	11.3
Dispersion in MPK	3.3	5.5	8.5	4.8
Average capital stock	−7.8	−6.8	−8.1	−9.2
Total output	−1.6	−1.3	−2.3	−1.7
Wage rate	−2.1	−1.5	−3.0	−2.4
Mass of owners	−1.8	−0.6	−2.6	−2.5
Average unit price	−22.1	−33.4	−14.1	−35.8
Deadweight loss	0.049	0.039	0.076	0.052

Notes: As in Table 9, but computed in the augmented model of this appendix, in which the income tax allows a depreciation deduction and the capital-gains tax is levied on the gain over basis $p^s - a$. All columns raise the same revenue as the U.S. system; entries are percent changes relative to the no-tax economy—whose allocations coincide with those under the ideal tax scheme, the benchmark of Table 9—except deadweight loss, which is reported per dollar of revenue using the definition of Table 9.

Aggregate tax revenue in steady state is the sum of the three instruments,

$$R \;=\; \tau_w \int w\, n(x)\, \phi(x)\, dx \;+\; \tau_b \int \big[\pi(x) \;-\; \delta_k a\big]\, \phi(x)\, dx \;+\; \tau_c\, \eta \int \big[p^s(x,\tilde{x}) - a\big]\, \lambda^s(x,\tilde{x})\, \phi(x)\, dx\, d\tilde{x}. \tag{47}$$

The first integral is the wage tax on the aggregate wage bill; the second is the per-period income tax aggregated over all owners; the third is the capital-gains tax aggregated over sellers, weighted by the equilibrium sale rate $\eta \lambda^s$.

No-tax limit Setting $\tau_b = \tau_c = 0$ in (45) eliminates the depreciation rebate and the seller's basis credit. The only remaining basis term is the drift $-\delta_k a\, \partial_a V$, which equals zero whenever $\partial_a V = 0$. The augmented HJB therefore admits a solution that is constant in a. The KFE (46) then has a stationary measure whose (z,k)-marginal coincides with the baseline distribution. The augmented equilibrium reduces to the equilibrium of Section 3 of the main text.

C.3 Results

We recompute the revenue-neutral reforms of Table 9 in the augmented model, letting the income tax deduct depreciation and the capital-gains tax fall on the gain over basis. Table 10 reports the

results. Most of the outcomes are similar to those in the baseline model, so we highlight the key differences. The fraction of capital traded drops by less when deductions lower the capital gains tax burden on sellers, and allow the buyers to partially deduct their purchases from income taxes. In regimes without capital gains taxes, depreciation deduction might even increase the fraction of capital traded with respect to the no-tax economy. Crucially, while distortions from taxation are quantitatively smaller, especially in the economies with positive capital gains taxes, the ranking of tax regimes is the same as in our baseline model in Section 8.