Local Labor Markets and Aggregate Productivity: Online Appendix

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A Equilibrium Values

A.1 The Value of Wage Contracts

Let $W(h_\ell, a, e, i, z, \omega)$ be the difference between the present discounted value of a worker of human capital h_ℓ, age a, education e, who lives in city i, is employed at match quality z, and is being paid a piece rate ω and her value if unemployed. That is, W is the portion of the surplus that accrues to the worker if the piece rate is ω. It is then convenient to highlight the discrepancy between W and the surplus of the match, S. The HJB equation that characterizes W can be written as

$$\begin{aligned} & rW(h_\ell, a, e, i, z, \omega) \\ &= rS - (1-\omega) z h_\ell \\ &+ [\sigma_i \kappa(G_i, h_\ell, e) + \eta^e exp(-\eta h_\ell)][(W(h_{\ell+1}) - W) - (S(h_{\ell+1}) - S)] \mathbb{1}\{a = y\} \\ &+ \psi_h[(W(o) - S(o))\mathbb{1}\{a = y\} - (W - S)] + \delta^e(S - W) + \mathcal{W}(i) + \mathcal{W}^*(i), \end{aligned} \tag{A.1}$$

where it is understood that individual state variables are omitted from the RHS if they take the same value as on the LHS.

The LHS is the annuitized value of W. The first line on the RHS has two terms. The first term is the annuitized surplus from the worker's current match. The second term captures the fact that the worker only receives a fraction ω of the flow payoff zh_ℓ, while $(1-\omega)zh_\ell$ accrues to the firm. The second line shows the change in value due to human capital accumulation, net of the change in the surplus of the match that is already included in the term rS. An analogous intuition applies to the aging process. The surplus of a match incorporates the event of job destruction, but workers only lose the portion of the surplus that they were receiving. Since firms lose $(S - W)$ when the job is destroyed, $\delta^e(S - W)$ needs to be added back to the worker's value. The last two terms represent the additional value the worker obtains when receiving outside offers and/or moving.

The value $\mathcal{W}(i)$ is given by

$$\mathcal{W}(i) = \lambda_{1,i} \left[(S - W)(1 - F(z)) + \int_{z^f(\omega)}^{z} (S(\hat{z}) - W) dF(\hat{z}) \right].$$

If a worker transitions to a new job within the same city, that is, to a job with match quality $\hat{z} > z$, she receives the portion of the surplus of the old match that was accruing to the previous employer before the workers' job-to-job transition (first term). This is because the previous employer is always willing to deliver, at most, the surplus of the match in order to retain the worker when competing with a poaching firm. The second term shows the gain from receiving an outside offer, inside the same city, that triggers a wage renegotiation but not a job-to-job transition. This event occurs when the worker and the new potential employer draw a match quality $\hat{z} < z$ such that $S(h_\ell, a, e, i, \hat{z})$ is higher than the current surplus to the worker, $W(h_\ell, a, e, i, z, \omega)$. The minimum match quality that triggers a wage increase is given by the value $z^f(h_\ell, a, e, i, z, \omega)$, which is implicitly defined by

$$S(h_\ell, a, e, i, z^f(h_\ell, a, e, i, z, \omega)) = W(h_\ell, a, e, i, z, \omega).$$

A worker who draws a match quality $z^f < \hat{z} < z$ receives the surplus of the new potential match, $S(\hat{z})$. This assumption is slightly different from Bagger et al. (2014), who use the bargaining protocol in Cahuc, Postel-Vinay, and Robin (2006). In Bagger et al. (2014), the worker would additionally receive a fraction β of the difference between the surplus of her current match and the new potential one.

The last term in Equation (A.1) shows the gain from meeting firms in city $-i$, and it is equal to

$$\begin{aligned}\mathcal{W}^*(i) = \lambda^*_{1,-i} \mathbb{E}_D[(S - W)\mathbb{1}\{c < x(z,0)\} + \\ (\Delta(i,-i,c) - W)\mathbb{1}\{x(z,0) < c < x(z,0,\omega\})]F(R(-i)) + \\ \lambda^*_{1,-i} \int_{R_{-i}}^{\bar{z}} \mathbb{E}_D[(S - W)\mathbb{1}\{c < x(z,\hat{z})\} + \\ (\Delta(i,-i,c) + S(-i,\hat{z}) - W)\mathbb{1}\{x(z,\hat{z}) < c < x(z,\hat{z},\omega)\}]dF(\hat{z}).\end{aligned} \tag{A.2}$$

The first two lines on the RHS of Equation (A.2) represents the situation in which the worker draws a match quality below the reservation value in city $-i$. The first term inside the expectation describes the case in which the worker draws a migration cost $c < x(z,0)$, hence she moves to city $-i$ and obtains the (additional) gain $S - W$. The second term is related to those draws that are both higher than the migration thresh-

old and sufficiently low that the worker can credibly threaten to quit and move unless her promised value is increased. The highest such cost, which equates the value from quitting to the current promised value satisfies

$$0 = \Delta(h_\ell, a, e, i, -i, x(h_\ell, a, e, i, z, 0, \omega)) - W(h_\ell, a, e, i, z, \omega).$$

The last two lines follow the same logic as the first two but applied to the case in which the new potential match quality $\hat{z}$ lies above the reservation value in city $-i$. In this circumstance, the maximum moving cost such that the threat to leave the current employer is credible, and the piece rate is renegotiated, satisfies

$$0 = \Delta(h_\ell, a, e, i, -i, x(h_\ell, a, e, i, z, \hat{z}, \omega)) + S(h_\ell, a, e, -i, \hat{z}) - W(h_\ell, a, e, i, z, \omega).$$

A.2 Firm Expected Surplus

Firms in city i receive a fraction $(1-\beta)$ of the expected surplus $\mathcal{S}(i)$ of creating a match with a worker. The expectation is taken with respect to the type of worker the firm meets, the match quality sampled by the firm-worker pair, and the migration cost—if the worker moves from city $-i$. The expected surplus in Equation (4.3) is then given by

$$\begin{aligned}\mathcal{S}(i) = \sum_{h_\ell, a, e} \int_{R_i}^{\bar{z}} \Big\{ & \phi(i,0) S(i,0,i,\hat{z}) + \mathbb{E}_{\phi(i,\tilde{z})}[S(i,\tilde{z},i,\hat{z})]\rho + \\ & \phi(-i,0)\mathbb{E}_D[S(i,0,i,\hat{z})\mathbb{1}\{c < x(-i,0,i,\hat{z})\} + \\ & (S(-i,0,i,\hat{z}) - c)\mathbb{1}\{x(-i,0,i,0) < c < x(-i,0,i,\hat{z})\}]\rho^* + \\ & \mathbb{E}_{\phi(-i,\tilde{z})}[\mathbb{E}_D[S(i,0,i,\hat{z})\mathbb{1}\{c < x(-i,\tilde{z},i,0)\} + \\ & (S(-i,\tilde{z},i,\hat{z}) - c)\mathbb{1}\{x(-i,\tilde{z},i,0) < c < x(-i,\tilde{z},i,\hat{z})\}]]\rho\rho^* \Big\} dF(\hat{z}).^1 \end{aligned} \tag{A.3}$$

The first line on the RHS corresponds to the surplus from hiring an unemployed (first term) or employed (second term) worker in the local labor market. Recall that the mass of unemployed workers in city i is equal to $\phi(i,0)$, while the equilibrium

[1] Once again, the state variables (h, a, e) are omitted from S and x.

distribution of employed workers over match quality $\tilde{z}$ is $\phi(i, \tilde{z})$. The parameter ρ in the second term captures the relative search efficiency of employed workers. The second and third line report the expected surplus from hiring a currently unemployed worker from city $-i$. If the worker draw a migration cost $c < x(-i, 0, i, 0)$, her bargaining threat point is the value of unemployment in city i. That is, from the point of view of bargaining with the firm, her migration cost is sunk. If instead $x(-i, 0, i, 0) < c < x(-i, 0, i, \hat{z})$ the worker's outside option consists in not moving, and the migration cost is factored into the bargaining process. The parameter ρ^* captures the relative search efficiency across cities. The interpretation of the last two lines follows the same logic as the two lines above but applied to hiring currently employed workers from city $-i$, who are distributed over match qualities according to $\phi(-i, \tilde{z})$.

B Motivating Evidence on Model Assumptions

B.1 Occupational and Educational Composition of Cities

I compute the size of the wage premium that can be accounted for by compositional differences between cities, and, specifically, by the overrepresentation in large cities of those workers with characteristics—specifically, education or occupation—that are correlated to higher wage growth.

In regression B.1, I project worker's i wage on indicators for living in a large city c, being a college graduate, and working in a given 3-digit census occupation, each interacted with the worker's age $a_i \in \{21\text{–}25, 26\text{–}30, 31\text{–}35, 36\text{–}40\}$. Table B.1 reports the estimated values of β_{large}, which are the key objects of interest. Column (1) shows the estimates of a regression without controls, or, equivalently, in which I restrict $\beta_{college,a} = \beta_{occ,a} = 0$. Columns (2) and (3) allow either $\beta_{college,a}$ or $\beta_{occ,a}$ to be different from zero, respectively. The main takeaway from this exercise is that the contribution of occupational sorting to the wage premium (reflected in the smaller coefficient β_{large} in Column (3) compared to Column (1)) is only slightly larger than that of education

Table B.1. Educational and Occupational Composition of Cities

Source: decennial census. In order to mimic the NLSY79 sample, I consider male workers between ages 20–30 (Census 1980), ages 26–35 (Census 1990), ages 31–40 (Census 2000). An indicator variable for each census year is included in the regression. The sample is further restricted to workers earning at least $\$5,000$ per year and \$1 per hour, working at least 10 hours per week and 45 weeks per year. I also exclude movers in "this or last year" for whom it is not possible to assign a CZ. All the standard errors (not reported) are of order 1e-3 and the p-values < 0.001.

$\beta_{large,a}$	(1)	(2)	(3)
large city × age 20–25	0.08	0.07	0.06
× age 26–30	0.18	0.15	0.13
× age 31–35	0.24	0.18	0.16
× age 36–40	0.26	0.19	0.17
control for education	N	Y	N
control for occupation	N	N	Y
R^2	0.090	0.158	0.268
n. of obs.	2.1M	2.1M	2.1M

(Column (2) versus Column (1)), which is explicitly modeled in this paper.

$$log(w_i) = \beta_0 + \beta_{large,a}\mathbb{1}\{c_i \text{ is large}\} \times a_i + \beta_{col,a}\mathbb{1}\{\text{college}\}_i \times a_i + \beta_{occ,a}\mathbb{1}\{\text{occupation}\}_i \times a_i + \varepsilon_i \tag{B.1}$$

B.2 City Size and City Composition

Some might wonder whether the random nature of the knowledge diffusion process presented in this paper should be replaced by a view in which college graduates a) only learn from each other and b) are perfectly able to locate each other within a city. In such a scenario, the wage—and especially wage growth—experienced by college graduates would be unaffected by the number of high school graduates that live in their same city. To test this hypothesis, in B.2 I regress the wage of a college

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	$\beta_{pop,a}$		$\beta_{col\ share,a}$
log(pop) × age 20–24	0.032	log(col share) × age 20–24	0.045
	(0.007)		(0.032)
× age 25–29	0.055	× age 25–29	0.081
	(0.003)		(0.016)
× age 30–34	0.068	× age 30–34	0.141
	(0.004)		(0.025)
× age 35–39	0.070	× age 35–39	0.186
	(0.005)		(0.034)
n. of obs.	357684	$R^2 = 0.124$	

graduate, indexed by i, on city size, $log(pop_{c_i})$, and on the fraction of city population with a college degree, $log(col\ share_{c_i})$. Both variables are interacted with the worker's age $a_i \in$ {21–25, 26–30, 31–35, 36–40}.

The results in Table B.2 show that college graduates experience higher wage (growth) in larger cities, but even more so in cities in which there is a high concentration of college graduates. Notice that $col\ share_{c_i}$ is equal to the ratio between the sheer number of college graduates and city size. Hence, college graduates of age a earn $(\beta_{pop,a} - \beta_{col\ share,a})$% higher wages in a city that is 1% larger but has the same number of college graduates. It is easy to see that $\beta_{pop,a} - \beta_{col\ share,a}$ is *always negative and strictly decreasing in a*. While it is reasonable to think that workers are able to partially direct their search for peers, the findings in this section speak in favor of the existence of a random component in the knowledge diffusion process.

$$log(w_i) = \beta_0 + \beta_{pop,a} log(pop_{c_i}) \times a_i + \beta_{col\ share,a} log(col\ share_{c_i}) \times a_i + \varepsilon_i. \qquad \text{(B.2)}$$

C Data

The representative cross-sectional sample of the NLSY79 includes 3003 men. I exclude workers who never entered the labor force, who were not employed for more than five consecutive years, or who were already in the labor force when they started being surveyed. Workers enter the labor force the first quarter in which they spend 390 hours either employed or unemployed, where the number of hours spent as unemployed is equal to the number of weeks of unemployment multiplied by 20. Once they enter the labor force, workers enter the estimation sample the first time they are employed. To keep a balanced panel but also avoid issues of non-random sample selection, I keep only workers that by 2012 had worked 20 full years since entering the estimation sample.

I build a monthly panel by sampling the interview week, whenever available, and the third week of the month otherwise. For each monthly observation, I observe labor market status, working hours, hourly wage, and whether the worker experienced a change of employer. I keep only wage observations that are associated with jobs at which workers spend at least 10 hours per week. I consider workers as still employed at their last job if they are observed not to be working for a certain period of time but then return to their last employer. In fact, it would be hard to justify the existence of search frictions and unknown match quality with the most recent past employer.

Location information, in the form of county of residence, is available at interview dates, and between interviews during 1978–1982 and after 2000. When location is not observed, I adopt the following assignment procedure. I assume that workers stay at their current location for the entire spell of a job, and I assign each job to the modal location in case I observe more than one location for the same job. I assign each county to a commuting zone (CZ) using the cross-walk provided by David Dorn.[2] I assign an observation to the CZ observed in the previous month if such CZ is the same as the CZ where the worker lives one or two months after the missing observation. The remaining periods with missing location information are split into two parts: the first half is assigned to the last observed CZ and the second half to the first CZ observed af-

[2] https://www.ddorn.net/data.htm

ter the period with missing information. The underlying assumption is that the actual migration date is uniformly distributed over the period in which location information is not available. Last, I assign CZs to the "large city" category if they have a population of more than 750,000 in 1990, which guarantees both substantial heterogeneity in size between large and small cities and a comparable number of observations from each category in the NLSY79.

The final sample has 240 monthly observations for each of the 386 (1146) workers with a college (high school) degree.

D Additional results

Table D.1. Non-Movers Sample: Empirical and Model-Generated Moments

Targeted Moments Computed on the Set of Non-Movers		
Moment (large/small)	Data	Model
Labor Market		
unemp. rate (%)	5.1/5.4	5.0/5.0
n. of jobs	6.38/6.14	6.63/6.47
mean wage gap (%)	32.7	34.1
EE wage growth (%)	11/11.8	10.9/11.5
return to tenure (%)	22.5	25.8
college share (%)	31/19	31/20
Wage growth (%)		
bottom-col-large	6.6	6.8
bottom-col-small	4.8	4.6
top-col-large	4.0	4.0
top-col-small	2.5	3.0
bottom-hs-large	4.3	4.4
bottom-hs-small	3.8	3.1
top-hs-large	2.5	2.8
top-hs-small	1.3	2.0
wage 11–20 vs. 1–10 yr. of exp.	30.6	32.0
Initial Wage		
col wage premium (%)	41	38
$75^{\text{th}}/25^{\text{th}}$ pctile, hs	1.65	1.64
$75^{\text{th}}/25^{\text{th}}$ pctile, col	1.86	1.95

Figure D.1. Life-Cycle Wage Profiles in Large and Small Cities

Wage profile of workers in the sample according to their education (top versus bottom panels), city (left versus right), and position in the wage distribution in the first year of employment (top versus bottom lines inside each subplot). Data: dotted line. Model: solid line. The shaded area represents the 95% empirical bootstrap confidence interval.

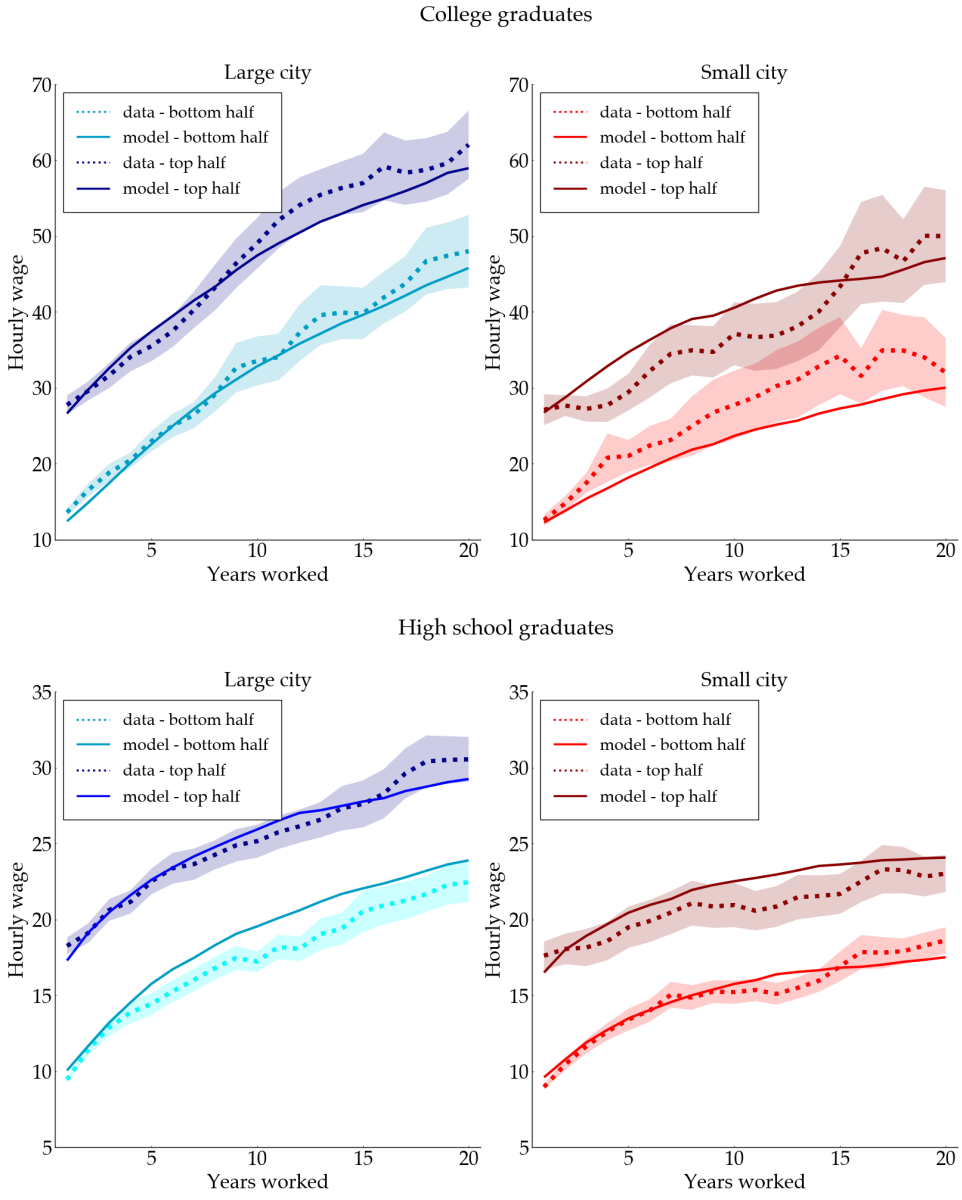
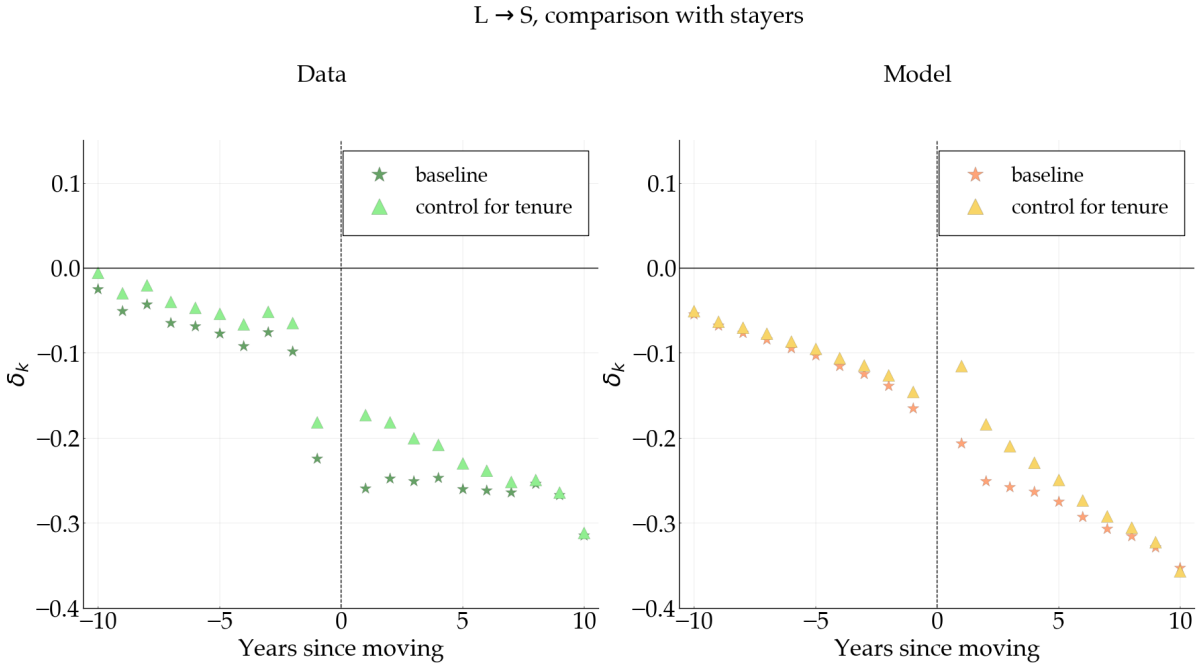


Figure D.2. Job Tenure and Migration

Wage of movers to small cities versus stayers in large cities. Dark stars: baseline. Light triangles: control for tenure. Left: data. Right: model.



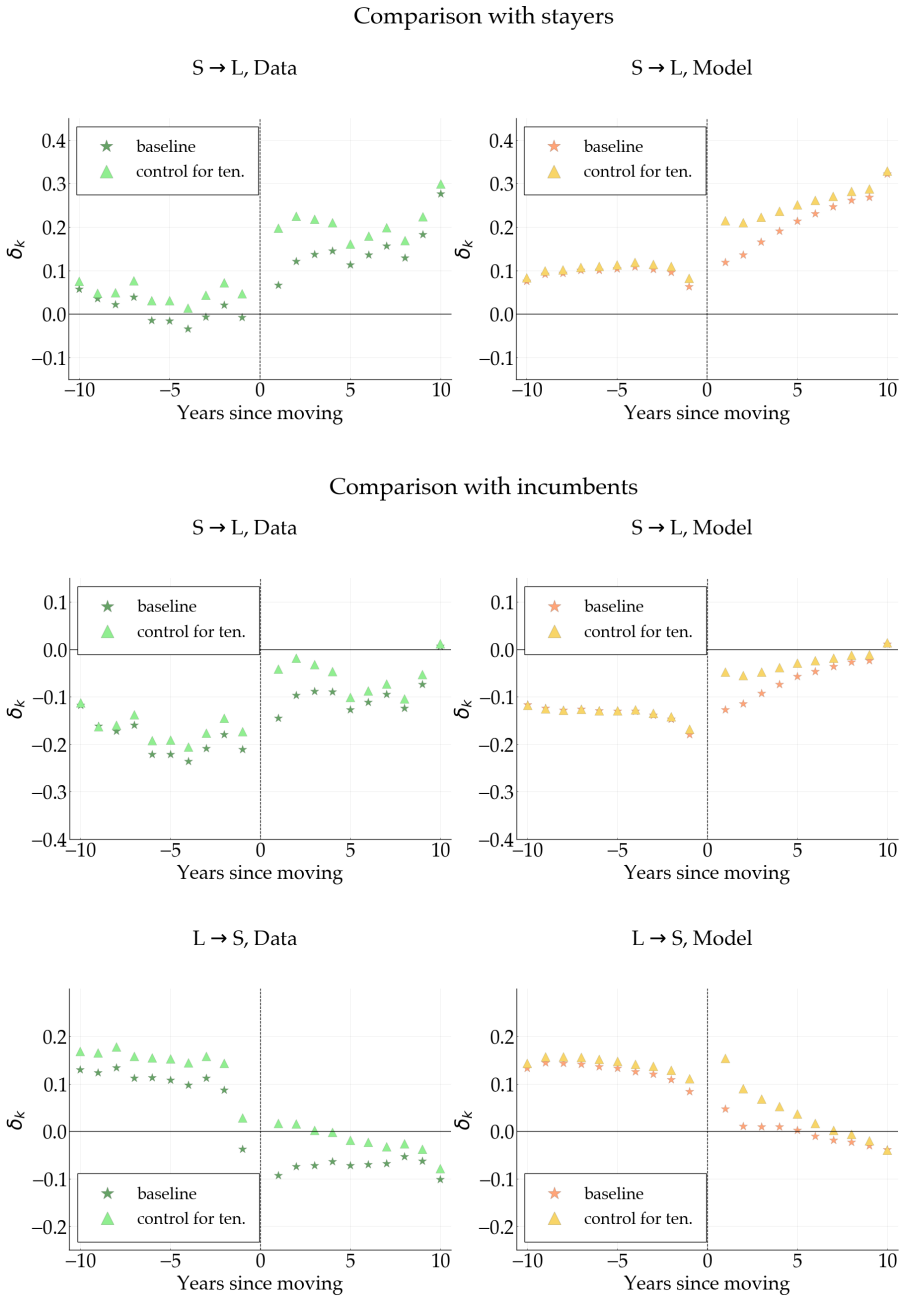
Job Tenure and Migration

In both the data and the model, there is a positive correlation between job tenure and wages. Recent migrants are relatively new at their job; hence, part of the discrepancy between their wage and non-movers' is, in fact, due to heterogeneity in job tenure. To illustrate this point, I augment Equation (3.3) in the text with a quadratic term in the tenure at the current job and compute new values for the coefficients δ_k. The updated plots highlight how the results differ from the baseline specification, but they do so in a very similar fashion in the model as in the data. As an example, Figure D.2 reports the comparison between movers to small cities and stayers in large cities. The left panel shows the coefficients estimated on NLSY79 data. While the relative wage loss after leaving a large city is essentially flat in the baseline regression (dark green stars), it is increasing over the years when tenure is controlled for (light green triangles). A very similar pattern emerges from the model (right panel, orange

shades). The intuition is simple. Since recent movers have relatively short tenure at their new job, controlling for job tenure reduces the size of the wage gap with respect to stayers, but only in the years right after migration. In subsequent years, the gap between stayers and movers gradually increases, as the latter no longer benefits from the superior learning opportunities in large cities. A similar logic applies to the other three panels reported in Figure D.3 below.

Figure D.3. Job Tenure and Migration (*continue*)

Wage of movers with respect to stayers (top row) and incumbents (bottom two rows). Dark stars: baseline. Light triangles: control for tenure. Left: data. Right: model.



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